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# Polarization squeezing with cold atoms

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We study the interaction of a nearly resonant linearly polarized laser beam with a cloud of cold cesium atoms in a high finesse optical cavity. We show theoretically and experimentally that the cross-Kerr effect due to the saturation of the optical transition produces quadrature squeezing on both the mean field and the orthogonally polarized vacuum mode. An interpretation of this vacuum squeezing as polarization squeezing is given and a method for measuring quantum Stokes parameters for weak beams via a local oscillator is developed.

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In optical communications and metrology, the precision of a measurement is mainly limited by quantum fluctuations of the light. However, it is well known that the standard quantum limit is not a fundamental one and for a few years a variety of methods to circumvent it generating squeezed states of the electromagnetic field have been developed in the quantum optics field (see for example [1] for a revue and references therein).

Recently a great deal of attention has been given to the quantum features of the polarization states of the light, essentially because of their connections with the quantum information technology. Several theoretical schemes to produce polarization squeezing using Kerr-like media have been proposed [2]. Experimental realizations achieve polarization squeezing by mixing squeezed vacuum (issued from an OPO) with a strong coherent beam on a polarizing beam splitter [3] or mixing two independent quadrature squeezed beams (issued from OPA) on a polarizing beam splitter [4]. Very recently it has been proposed to propagate a linearly polarized light beam through an atomic medium exhibiting self rotation to generate squeezed vacuum in the orthogonal polarization [5]: this is equivalent to achieving polarization squeezing. In this paper we focus on the theoretical and experimental investigation of polarization squeezing via the interaction of a nearly resonant linearly polarized laser beam with cold cesium atoms in a high finesse optical cavity. We describe the interaction between light and the atomic medium by means of an X-like four-level quantum model based on the linear input-output method. Our theoretical analysis shows clearly that, taking into account the atomic noise contribution due to the optical pumping, the self rotation (SR) term is unable to produce vacuum squeezing: cross-Kerr effect has to be included to recover it. Polarization squeezing is predicted by the model. Experimentally we observe a polarization squeezing of 13% and we develop a technique allowing the measurement of the quantum Stokes parameters for very weak beams, using a local oscillator (LO).

In previous works [6] we studied the interaction between a cloud of cold cesium atoms placed in a high finesse op-

tical cavity and a circularly polarized laser beam nearly resonant with an atomic transition. In such a situation, due to the optical pumping and to the presence of a closed transition, the atomic medium is conveniently modelled by an ensemble of two-level atoms. The saturation of the optical transition gives rise to an intensity-dependent refraction index. It is well known that the interaction of

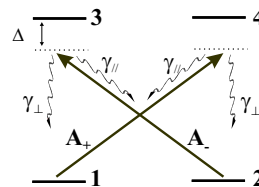


FIG. 1: Schematic energy level diagram for the X-like four-level system:  $\gamma_{\perp} + \gamma_{\parallel} = \gamma$  is the optical dipole decay rate;  $A_{\pm}$  denote the left- and right-circularly polarized components of the light field;  $\Delta$  is the detuning from resonance.

the light with a Kerr-like medium produces bistable behavior of the light transmitted by the cavity and that, at the turning point of the bistability curve, the quantum fluctuations of the light can be strongly modified and generate quadrature squeezing. Noise reduction up to 40% has been observed in our group [6].

In this paper we deal with a linearly polarized light beam interacting with the atomic ensemble: in such a configuration the two-level atom model is no longer applicable and the situation becomes more complicated. In the experiment the linearly polarized laser beam is nearly resonant with the  $6S_{1/2}$ ,  $F = 4$  to  $6P_{3/2}$ ,  $F=5$  transition: in principle, all the 20 Zeeman sub-levels are involved in the interaction; a rough estimation gives approximately the same amount of populations on all the sub-levels.

In order to get a qualitative physical insight into this problem, avoiding too heavy calculations, we model the atomic medium as an X-like four-level system (see Fig. 1). In such a system the competitive optical pumping between the  $\sigma_{\pm}$  components of the field induces non linear circular birefringence which depends on the ellipticity of the incoming field: this is the so called SR effect. It has

been shown that SR can produce squeezed vacuum on the orthogonal polarization of a linearly polarized beam undergoing free propagation in this kind of atomic media [5]. On the other hand, when the atomic media is placed in a cavity driven by a linearly polarized field the SR effect is responsible for polarization switching [7, 8].

We will investigate the possibility to generate squeezing on the orthogonal polarized field. To this end, we derive the quantum Heisenberg-Langevin equations for an  $X$ -like system driven by an  $x$ -polarized field, in the optical pumping approximation for which the following inequalities are satisfied:  $\Delta \gg \gamma \gg \gamma_{\perp} s_x$ , where  $\gamma = \gamma_{\perp} + \gamma_{\parallel}$  is the decay rate of the optical dipole,  $s_x = 2g^2|A_x|^2/\Delta^2$  is the saturation parameter. The equations for the orthogonally polarized field  $A_y$  and the atomic orientation  $J_z$  read:

$$\frac{dA_y}{dt} = -(\kappa + i\delta_{cav})A_y + i\delta_0 A_y - \delta_{nl} A_x + \quad (1)$$

$$+ \frac{2\kappa}{\sqrt{T}} A_y^{in} + F_{A_y}$$

$$\frac{dJ_z}{dt} = -\gamma_p \left[ J_z - \frac{N}{2} \frac{\delta S_3}{|A_x|^2} \right] + F_{J_z} \quad (2)$$

where  $\kappa$  is the cavity decay rate,  $T$  is the intensity transmission coefficient of the cavity mirror,  $\delta_{cav}$  is the detuning introduced by the cavity,  $\delta_0$  is the detuning due to the atoms in absence of optical pumping,  $\gamma_p = \gamma_{\perp} s_x$  is the optical pumping rate,  $\delta S_3 = i(A_y^{\dagger} \langle A_x \rangle - \langle A_x \rangle^* A_y)$  is the fluctuating part of the usual Stokes parameter,  $N$  is total number of atoms,  $F_{A_y}$ ,  $F_{J_z}$  are the standard delta-correlated quantum Langevin forces,  $\delta_{nl} = -(2/N)\delta_0 J_z$  is the non linear detuning due to circular birefringence, proportional to the difference in the ground state populations. This fluctuating term, with zero mean value and linked to the fluctuations of the fields interacting with the atoms, is responsible for the self rotation experienced by the incoming linearly polarized light.

Since our purpose is to investigate the noise fluctuations in the frequency domain, we consider the Fourier-transformed of the equations (1) and (2). Note that the optical pumping approximation limits the analysis to a range of frequencies such that  $\omega \ll \gamma$ . After straightforward calculations, the SR term can be written as follows:

$$\delta_{nl}(\omega) A_x \simeq i\delta_0 \frac{\gamma_p}{\gamma_p - i\omega} (A_y - \frac{A_x^2}{|A_x|^2} A_y^{\dagger}) + \quad (3)$$

$$- \frac{2}{N} \delta_0 A_x \frac{F_{J_z}}{\gamma_p - i\omega}$$

The phase dependent term in equation (3) could, in principle, generate quadrature squeezing on the  $A_y$  mode. Unfortunately the last term, which accounts for atomic noise contributions, cancels completely the expected squeezing in both the good ( $\kappa \ll \gamma_p$ ) and bad ( $\kappa \gg \gamma_p$ ) cavity limits. On the other hand, it is clear from the same equation that both these competing effects disappear for

frequencies such that  $\omega \gg \gamma_p$ , therefore, in order to recover squeezing, we have to consider saturation effects. In the limit  $\omega \gg \gamma$  the equation for the  $A_y$  mode is:

$$-i\omega A_y = -(\kappa + i\delta_{cav})A_y + i\delta_0 A_y + \quad (4)$$

$$- \delta_0 \frac{s_x}{2} \left( 2A_y - \frac{A_x^2}{|A_x|^2} A_y^{\dagger} \right) + \frac{2\kappa}{\sqrt{T}} A_y^{in} + F_{A_y}$$

The cross-Kerr term due to the optical transitions saturation allows to generate vacuum squeezing in such a system. A similar equation is derived for the  $A_x$  mode which is in turn squeezed. In agreement with this analysis, the full quantum model based on linear input-output theory predicts squeezing on both the orthogonally  $x$ - and  $y$ -polarized field modes [8].

The configuration used in the experiment is described in detail in [9]. We present on Fig. 2 only the main features. The cesium atoms are cooled in a standard magneto-optical trap which operates with three orthogonal circularly polarized trapping beam generated by a Ti:Sapphire laser and an inhomogeneous magnetic field. The trapping Ti:Sapphire laser is detuned by 3 times the

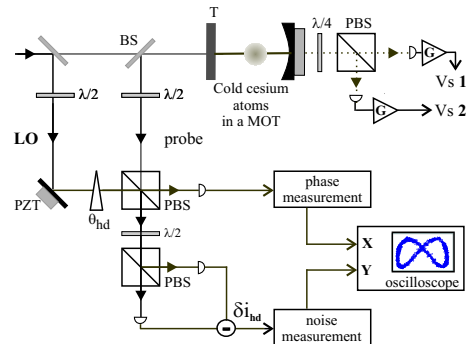


FIG. 2: Experimental set-up: PBS: polarizing beam splitter; BS: 10/90 beam splitter;  $\lambda/2$ : half-wave plate; PZT: piezoelectric ceramic.

linewidth of the upper state on the low frequency side of the  $6S_{1/2}$ ,  $F=4$  to  $6P_{3/2}$ ,  $F=5$  transition. To prevent atoms from being optically pumped to the  $6S_{1/2}$ ,  $F=3$  state, we superimpose a diode laser tuned to the  $6S_{1/2}$ ,  $F=3$  to  $6P_{3/2}$ ,  $F=4$  transition to the trapping beams. We use a 25 cm long linear cavity built around the cell (fig. 2). The cavity is close to the hemifocal configuration with a waist of  $260\mu\text{m}$ . The coupling mirror has a transmission coefficient of 10%, the rear mirror is highly reflecting. Hence, the cavity is close to the bad cavity limit for which the cavity linewidth (5 MHz) is larger than the atomic linewidth (2.6 MHz). We probe the atoms with a linearly polarized laser beam detuned of about 50 MHz in the red of the  $6S_{1/2}$ ,  $F=4$  to  $6P_{3/2}$ ,  $F=5$  transition. The optical power of the probe beam ranges from 5 to 15  $\mu\text{W}$ . The light transmitted by the end mirror of the cavity is sent through a quarter-wave plate

and a polarizing beam splitter, and is finally detected by two high-sensitivity balanced photodiodes. This configuration allows us to perform an accurate analysis of the polarization of the light transmitted by the cavity, which is strongly affected by the interaction with the atomic medium, as it was discussed above. In order to control the cavity length, a laser diode beam, locked in frequency very far from atomic resonance (1 GHz below the probe beam frequency), is sent through the cavity. The portion of this beam transmitted by the cavity and detected by the photodiodes is used to lock the cavity length. The probe beam reflected by the rear cavity mirror after interaction with the atoms is sent through a half-wave plate and a polarizing beam splitter. In such a way, setting an appropriate position of the half-wave plate we are able to mix alternatively the main polarization mode (along  $x$ ) or the orthogonal vacuum mode (along  $y$ ) with a powerful (10 mW) local oscillator beam (LO) and send them to a standard homodyne detection. By varying the relative phase of the LO with respect to the probe beam we can detect the noise features of all the quadratures of the field.

We will now present the experimental results: first of all we have studied the bistable behavior of our system. In Fig. 3 we perform the analysis of the polarization state for the light transmitted by the cavity as a function of its length.

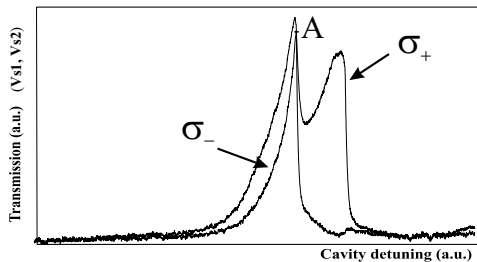


FIG. 3: Analysis of the circular polarization content of the light transmitted by the cavity and detected by photodiodes shown in Fig. 2. Polarization switching occurs at point A.

Starting from the left up to the point A, the polarization remains linear (nearly equal amount of circular polarized light on both the photodiodes), then it becomes circular. In the last case the atoms interact with a circular polarized light; the detection of the noise of the orthogonal mode reveals that is in a coherent vacuum state. More interesting is the first situation where, in agreement with the theoretical predictions, we observe quadrature squeezing on both the main mode (polarized along the  $x$  axis) and the orthogonal mode (along  $y$ ), which is therefore in a squeezed vacuum state. The best results are 5% (7% after correction for optical losses) of noise reduction for the main polarization mode and 13% (18% after correction) for the squeezed vacuum state, as shown in Fig. 4. The noise of the mode with orthogonal

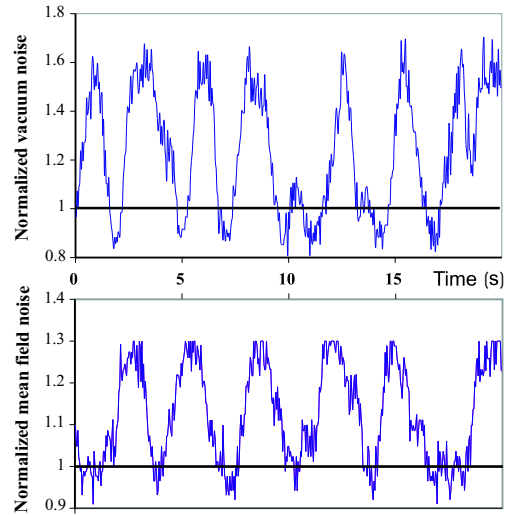


FIG. 4: Normalized quadrature noise for the vacuum field mode ( $A_y$ ) at 3 MHz and for the mean field ( $A_x$ ) mode at 5 MHz. The best squeezing is 5% for the mean field mode and 13% for the vacuum mode.

polarization with respect to the main mode is commonly referred as polarization noise: in fact a direct geometric interpretation of this noise is a jitter on the polarization direction of the main mode. In this sense the achieved vacuum squeezing is equivalent to the so called *polarization squeezing*. To give a complete characterization of the quantum features of the polarization state of the light under investigation, we have to measure the Stokes parameters of the beam. Following the literature [10], we define the quantum Stokes parameters as follows:

$$\begin{aligned}\hat{S}_0 &= \hat{A}_x^\dagger \hat{A}_x + \hat{A}_y^\dagger \hat{A}_y & \hat{S}_1 &= \hat{A}_x^\dagger \hat{A}_x - \hat{A}_y^\dagger \hat{A}_y \\ \hat{S}_2 &= \hat{A}_x^\dagger \hat{A}_y + \hat{A}_y^\dagger \hat{A}_x & \hat{S}_3 &= i(\hat{A}_y^\dagger \hat{A}_x - \hat{A}_x^\dagger \hat{A}_y)\end{aligned}\quad (5)$$

These operators obey the commutation relations  $[\hat{S}_0, \hat{S}_i] = 0$  ( $i = 1, 2, 3$ ) and  $[\hat{S}_i, \hat{S}_j] = \epsilon_{ijk} 2i\hat{S}_k$ . The spectral noise densities of these operators satisfy uncertainty relations  $V_{S_i}(\omega)V_{S_j}(\omega) \geq \epsilon_{ijk} \langle S_k \rangle^2$ . In general, the measurement of the variances of the Stokes parameters can be carried out directly by means of two balanced photodiodes and suitable combinations of half-wave and quarter-wave plates [10]. In our set-up, however, due to weak power of the probe beam interacting with atoms ( $\sim 10\mu W$ ), we need a powerful LO for the detection. In this case it is still possible to detect the Stokes parameters, as long as one knows the relative phase of the LO with respect to the mean field. In fact the expression giving the fluctuating part of the homodyne detection photocurrent with the  $A_y$  vacuum mode at the input can be expressed as a function of the fluctuating part of  $\hat{S}_2$  and  $\hat{S}_3$ :

$$\delta i_{hd} \propto \frac{|\alpha_{LO}|}{|\alpha_x|} (\cos \theta_{hd} \delta \hat{S}_2 + \sin \theta_{hd} \delta \hat{S}_3) \quad (6)$$

where  $\alpha_{LO}$ ,  $\alpha_x$  are respectively the LO and  $A_x$  amplitudes;  $\theta_{hd}$  is the relative phase between the LO and the mean field.  $\theta_{hd} = 0$  (respectively  $\pi/2$ ) corresponds to the detection of fluctuations of  $S_2$  (respectively of  $S_3$ ). Hence in the experiment we can get the Stokes parameters simply by measuring simultaneously the relative phase of the mean field mode and the LO and the quadrature noise of the  $A_y$  mode. This measurement is readily performed setting the half-wave plate before the mixing polarizing beam splitter (see Fig. 2) in such a way that the vacuum  $A_y$  mode is sent to the homodyne detection; automatically the mean field  $A_x$  goes through the other port of the beam splitter and is detected together with a portion of the LO by a photodiode. These two signals are displayed on the oscilloscope working in  $XY$  mode, giving the characteristic curves reported below. In Fig. 5 the

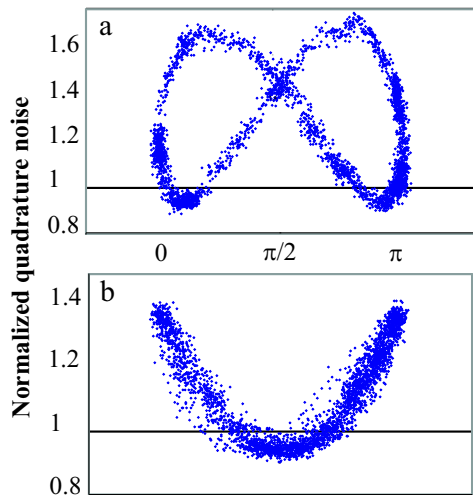


FIG. 5: Normalized quadrature noise for the  $A_y$  mode vs the relative phase between the mean field mode and LO. In curve (b), taken at a noise frequency of  $3MHz$ , quadrature squeezing is achieved when  $\theta_{hd} = \pi/2$ :  $\hat{S}_3$  is squeezed. The general case is shown in curve (a): a linear combination of  $\hat{S}_2$  and  $\hat{S}_3$  is squeezed.

normalized quadrature noise of  $A_y$  is plotted as a function of the relative phase between the mean field and the LO: in some special cases, depending on the chosen value of the noise frequency, when  $\theta_{hd} = 0$  or  $\pi/2$ , we can squeeze alternatively  $\hat{S}_2$  or  $\hat{S}_3$ . Simultaneous squeezing on both Stokes parameters is impossible because of the Heisenberg uncertainty relation. In general, for an arbitrary squeezed quadrature, a linear combination of  $\hat{S}_2$  and  $\hat{S}_3$  is squeezed.

We would like to stress that the generation of two orthogonally polarized quadrature squeezed beams with a pre-

cise phase relation is strictly equivalent to the realization of a pair of entangled beams. The formal demonstration of this and the measurement of the criterion derived by Duan et al. [12] for non-separability in continuous variables systems will be presented in a forthcoming publication [13].

To conclude, we have demonstrated that the resonant interaction of a linearly polarized laser beam with a cloud of cold atoms in an optical cavity can produce quadrature squeezing on the mean field mode and on the orthogonally polarized squeezed mode. We have shown that these results can be interpreted as polarization squeezing and developed a method to measure the quantum Stokes parameters for weak beams, using a local oscillator and a standard homodyne detection.

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- [1] L. Davidovich, *Rev. Mod. Phys.*, **68**, 127, (1996).
  - [2] A.S. Chirkin, A.A. Orlov, D.Yu. Paraschuk, *Kvant.Elektron.* **20**, 999 (1993) [*Quantum Electron.* **23**, 870 (1993)]; A.P. Alodjants, A.M. Arakelian, A.S. Chirkin, *JEPT* **108** 63 (1995); N.V. Korolkova, A.S. Chirkin, *J. Mod. Opt.* **43** 869 (1996).
  - [3] P. Grangier, R.E. Slusher, B.Yurke, A. LaPorta, *Phys. Rev. Lett.* **59**, 2153 (1987); J. Hald, J.L. Sorensen, C. Schori, E.S. Polzik, *Phys. Rev. Lett.* **83**, 1319 (1999).
  - [4] W.P. Bowen, R. Schnabel, H.A. Bachor, P.K. Lam, *Phys. Rev. Lett.* **88**, 093601 (2002).
  - [5] A.B. Matsko, I. Novikova, G.R. Welch, D. Budker, D.F. Kimball, S.M. Rochester, *Phys. Rev. A* **66**, 043815/1 (2002); I. Novikova, A.B. Matsko, G.R. Welch, private communication.
  - [6] A. Lambrecht, E. Giacobino, J.M. Courty, *Optics Comm.* **115**, 199, (1995); A. Lambrecht, T. Coudreau, A.M. Steinberg, E.Giacobino, *Europhys. Lett.* **36**, 93 (1996).
  - [7] E.Giacobino, *Optics Comm.* **56**, 249 (1985).
  - [8] V. Josse, A. Dantan, A. Bramati, M. Pinard, E. Giacobino, submitted to *J. Opt. B, Special Issue on Quantum optics and Quantum entanglement*, quant-ph/0302142.
  - [9] A.Z. Khoury, T. Coudreau, C. Fabre, E. Giacobino, *Phys. Rev. A*, **57**, 4397, (1998).
  - [10] N. Korolkova, G. Leuchs, R. Loudon, T.C. Ralph, C. Silberhorn, *Phys. Rev. A*, **65**, 052306 (2002).
  - [11] N. Korolkova, Ch. Silberhorn, O. Glockl, S. Lorenz, Ch. Marquardt, G. Leuchs, *Eur. Phys. J. D*, **18**, 229 (2002).
  - [12] L. M. Duan, G. Giedke, J. I. Cirac, P. Zoller, *Phys. Rev. Lett.*, **84**, 2722 (2000).
  - [13] V. Josse, A. Dantan, A. Bramati, M. Pinard, E. Giacobino, to be published.