

Appendix to Strategies in Sentential Reasoning

(by Jean-Baptiste Van der Henst, Yingrui Yang, and P.N. Johnson-Laird)

This appendix presents protocols of what the participants said, wrote, and drew as they carried out various inferential problems. It supplements the protocols reported in the paper, presenting examples of the following: 1. changes in mid-problem from one strategy to another, 2. the incremental diagram strategy, 3. the step strategy, 4. the compound strategy, 5. the chain strategy, and 6. and an unclassifiable protocol.

Each protocol gives the verbatim protocol apart from filled pauses and repeated words, but the protocols are abbreviated in two ways. First, each protocol uses color terms only to refer to the items to which the participants referred, e.g. "red" to refer to "the red marble", "red marble", or "red". Second, each protocol uses the following abbreviations: "iff" for "if and only if", "ore" for "or else". Where participants used only the initial letter of a color term, we present the full color name for clarity. Comments are within square brackets.

1. Protocols illustrating a change of strategy in the middle of a problem

A change from the compound strategy to the chain strategy (participant 4 in Experiment 1)

Problem 12:

Green ore gray.
 Gray iff white.
 White ore pink.
 If not green then pink? [Given conclusion]

Verbal protocol:

Green ore gray.
 It's one or the other.
 Gray iff white.
 So, gray and white are interrelated.
 White ore pink.
 If not green then pink.
 If not green then gray and white.

 If have a white ... Oh, wait!
 If not green then gray.
 There can only be a gray if there is a white.
 But, I can only have white or pink.
 Since I have white, I cannot have pink.

Diagrams and actions:

Circles "green", "or", "gray" in first premise.
 Circles "iff" in second premise.

 Circles "white", "or", "pink" in third premise.

 [Compound inference]
 Crosses out "green" in first premise.

 [Start of chain]
 [Second link in chain]
 [Third link in chain]

No. [Rejects conclusion]
 If there is no green ...
 If green then pink.
 Yes, it does not follow.

Comments: The participant makes a compound inference, and then switches to the chain strategy to construct a chain of conditionals leading from the antecedent of the conditional conclusion to the negation of its consequent.

A change from the compound strategy to the incremental diagram strategy (participant 9 in Experiment 1)

Problem 13:

White iff green.
 Green iff brown.
 Brown ore black.
 Black ore blue.
 If white then blue? [Given conclusion]

Verbal protocol:

White iff green.
 Green iff brown.
 Brown ore black.
 If brown then green and white.

 Black ore blue.

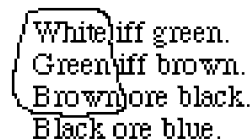
Brown ore black.
 Black ore blue.

If white then blue.
 Yes. [Accepts conclusion]
 If white then necessarily green and brown.
 If brown then not black.
 If black then blue. ["not" omitted]

Diagrams and actions:

[Compound inference from first two premises]

Draws circle round premises:



White iff green.
 Green iff brown.
 Brown ore black.
 Black ore blue.

Points to previous diagram.

Comments: The participant starts by drawing a compound conclusion, and then switches to drawing an incremental diagram, using circle to surround all the colors in one possibility.

2. Protocols illustrating the incremental diagram strategy

Incremental diagram used to evaluate a given conclusion (participant 5 in Experiment 1)

Problem 6:

Blue iff white.

White ore red.

Red iff pink.

If blue then pink? [Given conclusion]

Verbal protocol

Blue iff white.

Blue iff white.

White ore red.

White and blue, ore red.

Red iff pink.

Red and pink, ore white and blue.

If blue then pink. [the conclusion]

No.

Diagrams:

White

↓

Blue

White | Red

↓ |

Blue |

White | Red ← Pink

↓ |

Blue |

Comments: The participant represents the first premise by constructing a single possibility. He then updates this diagram after reading each of the premises. Finally, he rejects the given conclusion presumably because it is false in his diagram of the two possibilities consistent with the premises.

Incremental diagram used to draw a conclusion (participant 7 in Experiment 3)

Problem 7:

White or else red.

Red iff green.
Green ore yellow.

Verbal protocol

White ore red.
Red iff green.
Green ore yellow.

Diagrams:

1: White
2: Red

If red then green and not yellow.

1: White
2: Red green ~~yellow~~

If white then no red and no green and yellow.

1: White, ~~green~~ yellow
2: Red green ~~yellow~~

Either of these two cases. [Conclusion]

Comments: The participant represents the first premise by constructing two possibilities. He then considers separately each of these possibilities and updates the diagram by inferring what follows from each possibility.

Incremental diagram used to respond “nothing follows” (from participant 11 in Experiment 3).

Problem 10:

Black iff white
White or pink or both.
Pink iff purple.
What if anything follows?

Verbal protocol:

Black iff white

Diagrams:

White
|
Black

White or pink or both

White ----- pink
|
Black

Pink iff purple.

Purple
|
White ----- pink
|
Black

Nothing follows, you are not sure

if white and purple are together or not.

Comments: The participant constructs a diagram of the first premise. She then tries to integrate the content of the second premise. However, because it is an inclusive disjunction and not an exclusive one, she is not able to specify the relation between the white marble and the pink one and chooses to represent this relation with dashes. She fails to make a clear representation of the three possibilities compatible with the premises. As a result of her uncertainty about the relation between the white marble and the pink one, she thinks that there is no possibility of inferring a conclusion about the items in the left part of the diagram and those in the right part. Hence, she concludes erroneously that nothing follows from the premises.

3. Protocols illustrating the step strategy

The step strategy used to evaluate a given conclusion (participant 10 in Experiment 1)

Problem 8:

Black iff pink.
 Pink ore brown.
 Brown iff gray.
 If pink then gray? [given conclusion]

Verbal protocol:

Black iff pink.
 Pink ore brown.
 Brown iff gray.
 If pink then gray.
 I am assuming we have the pink.
 Then there is no brown.
 Then there is no gray.
 And there is black.
 Not gray because not brown.
 If pink then gray.
 No. [rejects conclusion]
 Because pink, not brown.
 With brown gone, then there is no gray.
 No. [rejects conclusion]

Diagrams and actions:

Underlines 'pink' in second premise.
 Crosses out brown in second premise.
 Crosses out gray in third premise.

Comments: The participant makes a supposition corresponding to the antecedent of the conclusion, and then infers a sequence of consequences culminating in the negation of the

consequent of the conclusion. This suffices for the rejection of the conclusion. The inference that black is a consequent of the assumption is valid, but irrelevant to the conclusion.

The step strategy used to generate a conclusion from a categorical premise (participant 17 in Experiment 3).

Problem 2:

White and blue.
 Blue ore gray.
 Gray ore pink.

Verbal protocol:

White and blue.
 Blue ore gray.
 Gray ore pink.
 White and blue.

Blue.
 So no gray.
 No gray.
 So pink.
 White, blue and pink. [Conclusion]

Comments: The participant does not write or draw anything, but starts with one of the conjuncts from the first premise and then derives a series of conclusions. The conclusion conjoins each of the conclusions in the series.

The step strategy used to generate a complex conditional conclusion from a single supposition (participant 2 in Experiment 3).

Problem 6:

White iff brown.
 Brown ore yellow.
 Yellow iff blue.

Verbal protocol:

White iff brown.
Brown ore yellow.

If brown ...
Then white because of the first sentence.
Cannot have a yellow because of the second sentence.
No blue because of the third sentence.
If brown, then white, no yellow, no blue. [Conclusion]

Comments: The participant does not write or draw anything, but makes a supposition based on the consequent of the first premise, and then derives a series of intermediate conclusions. Her final conclusion is a complex conditional in which the antecedent corresponds to the supposition and the consequent corresponds to a conjunction of the intermediate conclusions.

The step strategy used to draw two conditional conclusions from two suppositions (participant 17 in Experiment 3).

Problem 5:

Green ore blue.
Blue iff black.
Black iff red.

Verbal protocol:

Green ore blue.
Blue iff black.
Black iff red.
Green ore blue.
Blue iff black.
Black iff red.

Actions:

If green ...	Points to first premise.
So not blue.	Points to first premise.
No black.	Points to second premise.
No red.	Points to third premise.

If blue ... Points to first premise.
 No green.
 Black.
 Red.
 If green then no blue, black, or red. [First conclusion]
 If blue then there is a black and a red but no green. [Second conclusion]

Comments: The participant makes two suppositions on the basis of the exclusive disjunction. Her conclusions are complex conditionals in which the antecedent corresponds to the supposition and the consequent corresponds to a conjunction of the intermediate conclusions.

The step strategy used to formulate a modal conclusion (participant 3 in Experiment 3).

Problem 12:

Gray or yellow or both
 Yellow ore black.
 Black iff green.

Verbal protocol:

Gray or yellow or both.
 Yellow ore black.
 Black iff green.

If green . . .
 then black.
 No yellow.
 If green no yellow, may be gray we don't know. [Conclusion]
 If green no yellow, may be gray we don't know. [Repeats conclusion]

Comment: The participant infers what follows from the supposition “if green”, but is unable to draw a definite inference about the gray and accordingly states a modal conclusion.

4. Protocols illustrating the compound strategy

The compound strategy used to evaluate a given conclusion (participant 1 in Experiment 1)

Problem 10:

Pink iff red.

Red iff blue.

Blue ore white.

Pink ore white? [Given conclusion]

Verbal protocol:

Pink iff red.

Red for pink.

Red iff blue.

Blue for red.

Blue for pink. [Compound inference]

Blue ore white

Pink ore white. [Compound inference]

Pink only if blue. [Compound inference]

Blue ore white.

Pink needs blue. [Repeats it]

Not possibly blue if white.

Pink only if blue. [Repeats it again]

Pink only if blue.

Not possibly white for blue.

Yes: Pink ore white. [Compound inference]

Diagrams and actions:

Red \rightarrow Pink

Blue \rightarrow Red

Blue \rightarrow Pink

Points to previous diagram.

Points to previous diagram.

Points to previous diagram.

Blue \rightarrow Pink

~~White~~ \rightarrow Pink

Comments: The participant reformulates the first two premises, diagrams them, and draws a compound inference from them. Next, the participant uses this conclusion and the third premise to infer the given conclusion. He repeats the first compound inference three times, and finally repeats the compound inference yielding the conclusion to be evaluated.

The compound strategy used to formulate a conclusion (participant 1 in Experiment 3).

Problem 6

White iff brown.

Brown ore yellow.

Yellow iff blue.

Verbal protocol:

White iff brown.

Brown ore yellow.

Yellow iff blue.

White iff brown.

If brown then white. [Reformulates premise]

Brown ore yellow.

Yellow iff blue.

If blue then yellow. [Reformulates premise]

If yellow then not white. [Compound inference from first and second premises]

If yellow or blue, then not white. [Compound inference from previous step and third
premise]

If yellow or blue, then not white.

The compound strategy used to formulate an incomplete conclusion (participant 13 in Experiment 3).

Problem 11:

Yellow or black or both.

Black iff green.

Green or else red.

Verbal protocol:

Yellow or black or both.

Black iff green.

Green or else red.	
If green then black.	[Reformulates premise]
No black, if red.	[Compound inference from second and third premises]
If red then no black.	[Conclusion]

Comment: The participant draws a compound inference from two premises only. He ignores the first premise and makes a final conclusion on the basis of this inference.

5. Protocols illustrating the chain strategy

The chain strategy used to evaluate a given conclusion (participant 1 in Experiment 1)

Problem 11:

Gray iff red.	
Red ore white.	
White iff blue.	
If not gray then blue? [Given conclusion]	
<u>Verbal protocol:</u>	<u>Drawings and actions:</u>
Gray iff red.	
Red for gray.	Red → Gray
Red ore white.	
Red ore white.	Red X White
White only if blue.	
Blue for white.	Blue → White
If not gray then blue.	
Red is there only if ...	Points to first diagram.
Gray only if red.	
If not gray then not red.	Points to first diagram. [Start of chain]
If not red then white.	Points to second diagram.
White comes from blue being there.	Points to second and third diagram. [End of chain]
If not gray then not red.	
If not red then white.	
Yes. [Accepts conclusion]	

Comments: The participant draws a diagram for each premise, and then makes a series of immediate inferences to construct a chain of conditionals leading from the antecedent to the consequent of the conditional. He briefly recapitulates the first two conditionals in the chain prior to accepting the conclusion.

The chain strategy used to evaluate a given conclusion (participant 4 in Experiment 1)

Problem 15:

Gray iff pink.	
Green ore black.	
Pink iff green.	
If brown then gray?	
<u>Verbal protocol:</u>	<u>Drawings and actions:</u>
Gray iff pink.	Gray = Pink
Gray equals pink.	
Green ore black.	Writes: "green or black".
Green or black.	
Pink iff green.	Pink = Green
Pink iff green.	
Black ore brown.	Writes: "black or brown".
Black or brown.	Circles both terms in conclusion.
If gray then brown.	[Start of chain]
If gray then pink.	
Gray and pink.	
If pink then green.	Crosses out "black" in "green of black ".
If green then not black.	Crosses out "black" in " black or brown".
If not black then brown.	[End of chain]
Yes.	[Accepts conclusion]

The chain strategy used to generate two conclusions from two chains (participant 15 in Experiment 3).

Problem 5:

Green ore blue.
Blue iff black.
Black iff red.
<u>Verbal protocol:</u>
Green ore blue.
Blue iff black.
Black iff red.
If green then no blue.
If no blue then no black.

If no black then no red.
 If green then no red. [Conclusion from first chain]

If blue then black.
 If black then red.

If blue then red. [Conclusion from second chain]
 If green then no red, if blue then red.

Comments: The participant constructs two chains in which each starts with the disjunct of the first premise. The participant draws two conclusions, corresponding each to one of the chains, but finally combines without using any clear connective.

The chain strategy used to generate a modal conclusion (participant 19 in Experiment 3).

Problem 10:

Black iff white.
 White or pink or both.
 Pink iff purple.

Verbal protocol:

Black iff white.
 White or pink or both.
 Pink iff purple.

If purple then pink.
 If pink then possibly white.
 If white possible then black.
 If purple then definitely a pink and it is possible to have white;
 if white then definitely a black. [Conclusion]

Comment: The participant constructs a chain from the third premise to the first. Two conditionals in the chain contain a modal qualification, which is drawn in the conclusion.

6. An unclassifiable protocol (participant 6 in Experiment 3)

Problem 12:

Gray or yellow or both.
Yellow ore black.
Black iff green.

Verbal protocol:

Gray or yellow or both.
Yellow ore black.
Black iff green.

If green then black. [Reformulates third premise]
Yellow ore black.
If green then black
Yellow ore black.
Gray or yellow or both.
Nothing follows.

General comment: The participant just repeats or reformulates the premises but he is not able to draw any inferences by combining premises. Hence, he answers that nothing follows.
