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Stochastic resonance in periodic potentials: realization in a dissipative optical lattice

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PACS. 42.65.Es – Stimulated Brillouin and Rayleigh scattering.

PACS. 32.80.Pj – Optical cooling of atoms; trapping.

Abstract. – We have observed the phenomenon of stochastic resonance on the Brillouin propagation modes of a dissipative optical lattice. Such a mode has been excited by applying a moving potential modulation with phase velocity equal to the velocity of the mode. Its amplitude has been characterized by the center-of-mass (CM) velocity of the atomic cloud. At Brillouin resonance, we studied the CM-velocity as a function of the optical pumping rate at a given depth of the potential wells. We have observed a resonant dependence of the CM velocity on the optical pumping rate, corresponding to the noise strength. This corresponds to the experimental observation of stochastic resonance in a periodic potential in the low-damping regime.

A particle trapped in a potential well constitutes a model useful for the understanding of a variety of phenomena. The extension to a periodically modulated double well potential including a stochastic force leads to a complex nonlinear dynamics, and allows to modelize a variety of phenomena ranging from geophysics [1, 2] to bistable ring lasers [3], from neuronal systems [4] to the dithering effect in electronics [5] and so on. Indeed such a system exhibits the phenomenon of *stochastic resonance* (SR [6, 7]): the response of the system to the input signal (the modulation) shows a resonant dependence on the noise level (the amplitude of the stochastic force), so that an increase of the noise strength may lead to a better synchronization between the particle motion and the potential modulation.

The phenomenon of stochastic resonance is not restricted to static double-well potentials driven by a periodic and a stochastic force, and new types of stochastic resonance have been demonstrated in various systems, as systems with a single potential well, bistable systems with periodically modulated noise, and many others [8–14]. In particular much attention has been devoted to the analysis of SR in *periodic potentials* [9–14]. Indeed, many different physical systems are described in terms of periodic structures, and it is by now well established that the noise plays a major role in the mechanisms of transport in periodic structures. For example, the study of the underdamped motion of a particle in a periodic potential showed that it is the interplay between inertial and thermal effects which determines the peculiar mechanical properties of certain metals [15, 16]. This is precisely the regime examined in this

work: we study the SR phenomenon by taking as spatially periodic system a dissipative optical lattice [17]. The laser fields create the periodic potential and produce the stochastic process of optical pumping. The friction for atoms well localized in a potential well is very small, so that inertial effects are important (low-damping regime). We report the experimental observation of stochastic resonance on the propagation modes of a dissipative optical lattice and give a complete theoretical account of the experimental findings.

The three dimensional periodic structure is generated by the interference of four linearly polarized laser beams, arranged in the so-called lin⊥lin configuration (Fig. 1) [17]. The resulting optical potential has minima located on a orthorhombic lattice and associated with pure circular (alternatively σ^+ and σ^-) polarization. The lattice constants, i.e. the distance (along a major axis) between two sites of equal circular polarization are $\lambda_{x,y} = \lambda/\sin\theta$ and $\lambda_z = \lambda/(2\cos\theta)$, with λ the laser field wavelength, and 2θ the angle between two copropagating lattice beams.

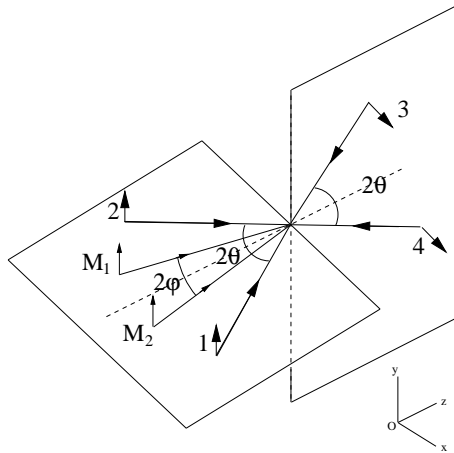


Fig. 1 – Laser fields configuration for the 3D lin⊥lin optical lattice. The beams 1 – 4 generate the static 3D periodic potential. Two additional laser beams (M_1 and M_2), are introduced to create a moving potential modulation.

The Brillouin-like propagation modes in such optical lattices have been first identified in Ref. [18] via semiclassical Monte Carlo simulations [19]. They consist of a sequence in which one half oscillation in a potential well is followed by an optical pumping process to a neighbouring well, and so on (Fig. 2). The velocity of the Brillouin mode is easily calculated by neglecting the corrections due to the anharmonicity of the optical potential. The time for an atom to do half an oscillation is then $\tau = \pi/\Omega_x$, where Ω_x is the x -vibrational frequency. This corresponds to an average velocity

$$\bar{v} = \frac{\lambda_x/2}{\tau} = \frac{\lambda\Omega_x}{2\pi\sin\theta}. \quad (1)$$

The direct observation of the Brillouin modes in optical lattices has been recently reported [20]. We note, however, that the detection scheme used in that work was based on the measurement of diffusion coefficients. These measurements require averaging of a large data set, and this makes difficult the exploration of a large interval of interaction parameters,

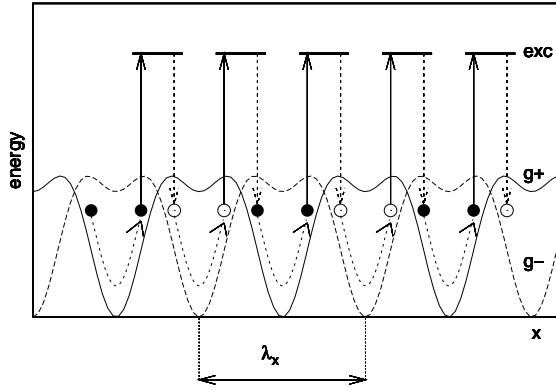


Fig. 2 – Atomic trajectory corresponding to a Brillouin mode in the x -direction. The shown potential curves (g_+ and g_-) are the section along $y = z = 0$ of the optical potential for a $J_g = 1/2 \rightarrow J_e = 3/2$ atomic transition and a 3D lin \perp lin beam configuration.

as necessary to evidence the phenomenon of stochastic resonance. The excitation scheme introduced in this work will result instead in significant variations of the atomic cloud center-of-mass motion, and leads to the observation of stochastic resonance, as described now.

The transport of atoms in optical lattices has been extensively studied [21–24]. In a *dissipative* optical lattice the dominant transport process is spatial diffusion [23, 24], and the Brillouin modes are greatly suppressed. To excite these modes it is necessary to create a potential modulation moving with phase velocity equal to the velocity of the Brillouin mode. This is done by introducing two additional y -polarized laser fields (M_1 and M_2 , see Fig. 1). They propagate in the xOz plane, symmetrically displaced with respect to the z -axis, and they form an angle equal to 2φ . These two modulation beams are taken to be sufficiently detuned from the lattice fields to neglect the interference between them and the lattice beams on the time scale of the atomic motion. In this way the modulation interference pattern is due only to the two fields M_1 and M_2 , and consists of an intensity modulation moving along the x axis with phase velocity

$$v_\phi = \frac{\delta_m}{|\Delta\vec{k}|} = \frac{\delta_m}{2k_m \sin \varphi} \quad (2)$$

where δ_m is the detuning between the fields M_1 and M_2 , and $\Delta\vec{k} = \vec{k}_{M_1} - \vec{k}_{M_2}$ the difference between their wavevectors ($|\vec{k}_{M_j}| \simeq k = 2\pi/\lambda$, $j = 1, 2$). This results in a moving modulation of the optical potential. For a $1/2 \rightarrow 3/2$ atomic transition⁽¹⁾, the modulated potential for the two ground states $|\pm 1/2\rangle$ reads

$$U_\pm(\vec{r}) = U_\pm^0(\vec{r}) + \delta U \cdot \cos[(\Delta k_x x - \delta_m \cdot t)] \quad (3)$$

with U_\pm^0 the optical potential of the unperturbed lattice

$$U_\pm^0(\vec{r}) = \frac{8\hbar\Delta'_0}{3} [\cos^2(k_x x) + \cos^2(k_y y) \mp \cos(k_x x) \cos(k_y y) \cos(k_z z)] , \quad (4)$$

⁽¹⁾It is customary in the analysis of Sisyphus cooling to consider a $1/2 \rightarrow 3/2$ atomic transition [17].

and $\delta U = 4\hbar\Delta'_{0,m}/3$ the amplitude of the potential modulation. Δ'_0 ($\Delta'_{0,m}$) denotes the light shift per lattice (modulation) field. We expect that for $v_\phi = \bar{v}$, i.e. for $\delta_m = \pm\Omega_B$, with

$$\Omega_B \equiv \frac{2 \sin \varphi}{\sin \theta} \Omega_x, \quad (5)$$

the Brillouin mode is excited, with the atoms following the potential modulation. This has been confirmed by Monte Carlo simulations. For a given modulated optical potential U_\pm and a given optical pumping rate Γ'_0 , we calculated the position of the center of mass (CM) of the atomic cloud as a function of the interaction time, for different values of the detuning δ_m between the two driving fields.

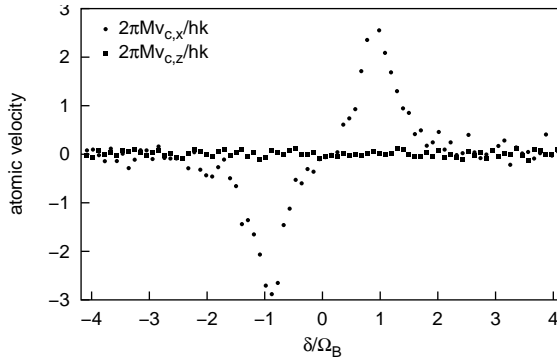


Fig. 3 – Numerical results for the velocity of the CM of the atomic cloud as a function of the detuning δ_m between the two driving fields. The velocity is in units of recoil atomic velocity v_r . The lattice beam angle is $\theta = 30^\circ$, the lattice detuning from atomic resonance $\Delta = -10\Gamma$ and the light shift per beam $\Delta'_0 = -200\omega_r$. Here Γ and ω_r are the width of the excited state and the atomic recoil frequency, respectively. The driving field angle is $\varphi = 10^\circ$, the detuning $\Delta_m = -30\Gamma$ and light shift per beam $\Delta'_{0,m} = -20\omega_r$.

The application of the moving modulation produces a motion of the CM of the atomic cloud. Its velocity v_c strongly depends on the velocity of the moving modulation, i.e. on the detuning δ_m between the driving fields, and shows two resonances centered at $\delta_m = \pm\Omega_B$ (Fig. 3). These resonances correspond to the excitation of the propagation mode in the $\pm x$ direction: at $\delta_m = \pm\Omega_B$ the velocity of the moving modulation is equal to the velocity of the Brillouin mode, and the atoms follow the potential modulation. On the contrary, for a velocity of the moving modulation very different from the velocity of the Brillouin mode ($|\delta_m| \gg \Omega_B$ or $|\delta_m| \ll \Omega_B$) the atomic dynamics is left unperturbed, and the CM of the atomic cloud does not move. This analysis shows that the effective excitation of the Brillouin propagation modes can be detected by observing a displacement of the CM of the atomic cloud. This will be the strategy followed in our experiment. We verified that the excitation of the Brillouin modes also leads to a resonant increase of the diffusion coefficient in the x -direction, in agreement with previous results for a different modulation scheme [20].

In our experiment, ^{87}Rb atoms are cooled and trapped in a magneto optical trap. The trapping beams and the magnetic field are then suddenly turned off. Simultaneously the four lattice beams are turned on. After 10 ms of thermalization of the atoms in the lattice the

two laser fields for the moving modulation are introduced according to the geometry of Fig. 1. The lattice angle is $\theta = 30^\circ$, while the two driving fields form an angle $2\varphi = 37^\circ$. The two driving fields are derived from an additional laser, with their relative detuning controlled by acousto-optical modulators.

The transport of the atoms in the optical lattice is studied by direct imaging of the atomic cloud with a CCD camera. We verified that for a given detuning δ_m , i.e. for a given velocity of the moving potential modulation, the motion of the center of mass of the atomic cloud is uniform and correspondingly determined the CM velocity v_c . By repeating the measurements for different detunings between driving fields, we obtained the x - and z -component of the CM-velocity v_c as a function of δ_m , as reported in Fig. 4. The x -component shows a resonant behaviour with the detuning δ_m , with two resonances of opposite sign symmetrically displaced with respect to $\delta_m = 0$. The position of these resonances is in agreement with the value $\Omega_B \simeq 2\pi \cdot 55$ kHz derived from the lattice parameters via Eq. (5). In contrast, the data for the z -component $v_{c,z}$, whose offset value corresponds to the radiation pressure of the modulation fields, do not show any resonance. These results are in agreement with our numerical simulations and constitute the direct experimental observation of the Brillouin propagation modes via the detection of the displacement of the CM of the atomic cloud.

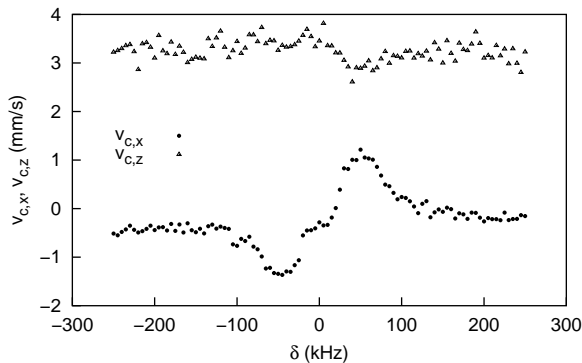


Fig. 4 – Experimental results for the velocity of the CM of the atomic cloud as a function of the detuning δ_m between driving fields. The lattice parameters are: lattice detuning $\Delta/(2\pi) = -45.6$ MHz, intensity per lattice beam $I = 2.3$ mW/cm², lattice angle $\theta = 30^\circ$. These parameters correspond to a vibrational frequency in the x - direction $\Omega_x/(2\pi) \simeq 45$ kHz. The parameters for the moving modulation are: $I_{M1} \simeq I_{M2} \simeq 0.5$ mW/cm², $\Delta_m/(2\pi) = -44$ MHz, $2\varphi = 37^\circ$. From these data we derive through Eq. (5) $\Omega_B \simeq 2\pi \cdot 55$ KHz, in excellent agreement with the experimental findings.

The Brillouin propagation modes are determined by the synchronization of the oscillations within a potential well with the hopping from a well to a neighbouring one produced by the optical pumping ⁽²⁾⁽³⁾. We studied the amplitude of the Brillouin mode, here characterized

⁽²⁾The propagation mechanism associated with these modes differs from that encountered in dense fluids or solid media. The atomic density is so low that the interaction between the different atoms is completely negligible, therefore the mechanism for the propagation of atoms cannot be ascribed to any sound-wave-like mechanism.

⁽³⁾A non-zero current in a symmetric periodic potential can also be obtained by modifying, through an external driving field, the activation energies of escape from a well, as described in [25, 26]. However that mechanism of directed diffusion does not correspond to the propagation of atoms at a well defined velocity, as

by the velocity of the CM of the atomic cloud $v_c(\delta_m = +\Omega_B)$ [analogous results are obtained for $v_c(\delta_m = -\Omega_B)$], as a function of the optical pumping rate Γ_0 for a given modulated optical potential. The numerical results display the SR-like nonmonotonic dependence of the amplitude of the Brillouin mode on the noise strength (Fig. 5), in agreement with our previous results for a different modulation scheme [20]. This SR-scenario has one important peculiarity with respect to the model usually considered in the analysis of stochastic resonance. Stochastic resonance is in general understood as the noise-induced enhancement of a weak periodic signal with a frequency much smaller than the intrawell relaxation frequency within a single metastable state. In contrast, in the present case, the noise synchronizes precisely with the intrawell motion of the atoms.

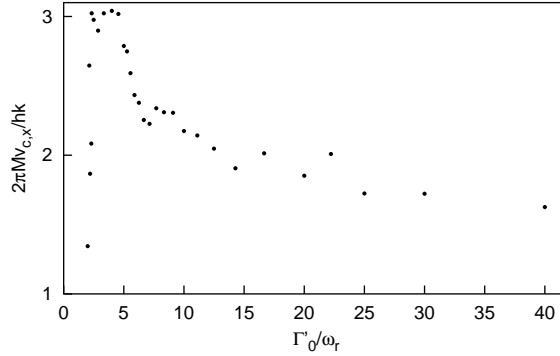


Fig. 5 – Numerical results for the x -component of the velocity of the center of mass of the atomic cloud as a function of the optical pumping rate, for a given depth of the optical potential wells. Parameters of the calculations are: $\theta = 30^\circ$ and $\Delta_0 = -100\omega_r$, $\varphi = 10^\circ$, $\Delta_m = -30\Gamma$ and $\Delta_{0,m} = -10\omega_r$.

Although $v_{c,x}(\delta_m = +\Omega_B)$ and $v_{c,x}(\delta_m = -\Omega_B)$ are expected to have the same dependence on the optical pumping rate, experimentally it is more convenient to characterize the amplitude of the propagation mode by the peak-to-peak amplitude ξ

$$\xi = v_{c,x}(\delta_m = +\Omega_B) - v_{c,x}(\delta_m = -\Omega_B) \quad (6)$$

of the CM velocity curve (as the one of Fig. 4). By doing so, the eventual uniform drift of the atomic cloud along the x - direction as a result of the radiation pressure deriving from a small difference in the driving fields intensities does not affect our measurements. We studied the ξ parameter at Brillouin resonance as a function of the optical pumping rate Γ'_0 at a given depth of the potential wells. This has been done by varying the lattice intensity I and detuning Δ so to keep the depth of the potential wells $U_0 \propto I/\Delta$ constant while varying the optical pumping rate $\Gamma'_0 \propto I/\Delta^2$. The intensity and the detuning Δ_m of the modulation fields are instead kept constant. Results of our measurements of ξ as a function of the optical pumping rate at a given depth of the potential wells and given modulation are shown in Fig. 6.

The typical behaviour of SR is observed: the parameter ξ increases with Γ'_0 at low pumping rates; then a maximum is reached corresponding to the synchronization between the optical pumping from one well to the next one with the oscillation in the potential wells; finally at larger pumping rates this synchronization is lost and ξ decreases.

in our case.

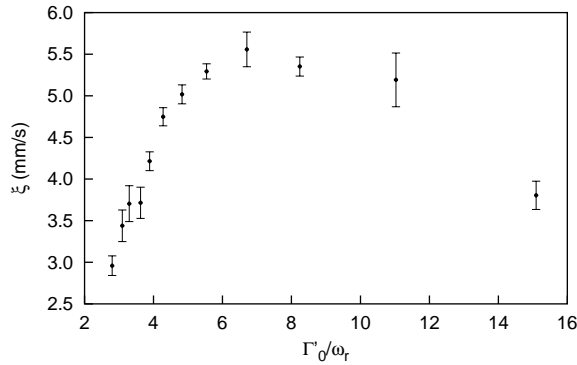


Fig. 6 – Experimental results for the peak-to-peak amplitude ξ of the CM velocity curve, as a function of the optical pumping rate Γ'_0 , at a given depth of the potential wells and given amplitude modulation. The light shift per lattice beam is $\Delta'_0 = -37.5\omega_r$. The parameters of the laser fields creating the moving intensity modulation are the same as for Fig. 4.

In conclusion, we reported the observation of stochastic resonance on the Brillouin modes of a dissipative optical lattice. These modes have been excited by applying a moving potential modulation with phase velocity v_ϕ equal to the velocity \bar{v} of the Brillouin mode. This results in a motion of the center of mass of the atomic cloud. The effective excitation of the Brillouin propagation mode has been detected by observing a resonant dependence of the velocity of the atomic cloud CM on the velocity of the moving modulation, with a maximum CM-velocity at $v_\phi = \bar{v}$. To observe the phenomenon of stochastic resonance in the optical lattice, we studied the CM-velocity at Brillouin resonance as a function of the optical pumping rate at a given depth of the potential wells and a given modulation amplitude. The SR-like nonmonotonic dependence of the CM-velocity on the optical pumping rate has been observed.

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