Combinatorix: A Tangible User Interface that Supports Collaborative Learning of Probabilities

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ABSTRACT
Teaching abstract concepts is notoriously difficult, especially when we lack concrete metaphors that map to those abstractions. *Combinatorix* offers a novel approach that combines tangible objects with an interactive tabletop to help students explore, solve and understand probability problems. Students rearrange physical tokens to see the effects of various constraints on the problem space; a second screen displays the associated changes in an abstract representation, e.g., a probability tree. Using participatory design, college students in a combinatorics class helped iteratively refine the Combinatorix prototype, which was then tested successfully with five students. Combinatorix serves as an initial proof-of-concept that demonstrates how tangible tabletop interfaces that map tangible objects to abstract concepts can improve problem-solving skills.

Author Keywords
Education; Probability; Tabletop; Tangible User Interfaces; Collaborative Learning.

ACM Classification Keywords
H.5.m. Information interfaces and presentation: User Interfaces.

General Terms
Human Factors; Design; Measurement.

INTRODUCTION
Many decisions benefit from understanding probability, e.g., when a patient must interpret the meaning of a medical test result or when a politician must weigh the costs and benefits of a particular policy. Unfortunately, Tversky and Kahneman [11] demonstrated that everyone, even professional statisticians, suffer from systematic biases in their intuitive judgements of probability. Students make a variety of identifiable mistakes when solving probability problems [1] and even graduate students who plan to teach mathematics retain strong misconceptions [6].

The challenge is how to help students develop an intuitive grasp of these abstract concepts. We are particularly interested in combinatorics, a branch of probability that deals with the enumeration, combination, and permutation of sets of elements and their mathematical relationships, because it results in a combinatorial explosion: even simple problems result in hundreds of possibilities that cannot be represented simply with physical objects, virtual or otherwise. Although some interactive tabletops offer a one-to-one mapping between physical objects and virtual concepts to support learning [2], there is a paucity of TUIs supporting learning of complex, *abstract* concepts.

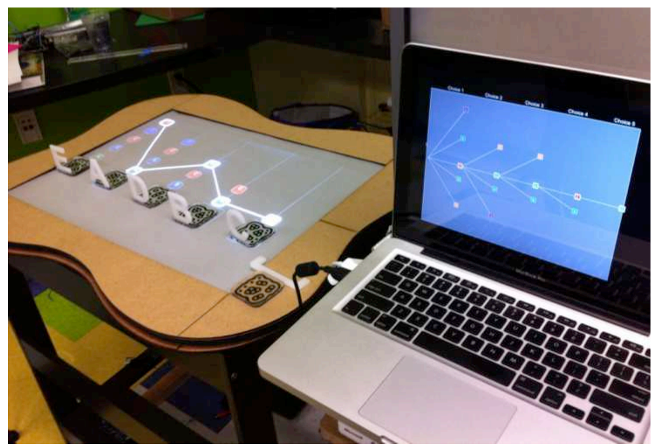

Figure 1. Combinatorix: Tangible objects control a tabletop display (left) and corresponding probability tree (right screen)

This paper describes the design and development of Combinatorix (Fig.1), a tangible tabletop interface in which users manipulate physical objects to obtain deeper insights into complex mathematical relationships. Our goal is not to transform virtual into physical objects but rather to use physical objects to explore fundamentally abstract concepts in combinatorics. We discuss the benefits and the challenges of our approach and conclude with an analysis of how tabletops and tangible user interfaces can affect education.

DESIGN PROBLEM
The original motivation for this project stemmed from observations of students in a university-level course in combinatorics. Faced with only paper and pencil, many had difficulty developing intuitions about probabilities [4] and suffered from the 'stereotype threat' [10] that they are poor in math. We hoped that letting students manipulate concrete objects while simultaneously observing the corresponding changes in deep structure, e.g. a probability tree, would reinforce their intuitions about the underlying mathematical

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principles. Our goal was to create an engaging and playful environment that avoids excessive mathematical notations and encourages discussion.

We support Fast's [3] constructivist approach, which emphasizes "overcoming misconceptions through supportive frameworks such as a series of anchoring situations". Our approach builds upon the "Preparing for Future Learning" (PFL) framework [9] in which students begin by analyzing contrasting examples of a concept to isolate important "deep features" of a combinatorics problem, in contrast to the "surface features" or superficial characteristics of a model [5]. Rather than limiting the number of cases, students should be able to express a variety of cases, each with their own visual representations.

Combinatorix is not designed to teach probability per se, but rather to provide a learning environment that encourages small groups of students to explore and discuss combinatorics problems. They should be able to express ideas and hypotheses, struggling with concepts in a productive way. Ideally, they will build their own theories, appreciate the challenges of defining an elegant formula and understand what their personal strengths and weaknesses are.

The learning environment should provide students with tools for reasoning about probabilities, including visualizations that support their reflections. Students should be able to associate common features of a problem with accepted mathematical representations, e.g., a probability tree. This implies that students need two interactive spaces: one for manipulating concrete, physical objects to explore the problem space and one for displaying the corresponding abstract representation of the problem space. The specific learning goals are to:

- learn the concepts of sample and event spaces, with probability defined as a ratio of the two;
- compute sample and event spaces using factorials, permutation and combinations with various constraints; and
- identify the deep structure of a problem as a probability tree and transfer this understanding to new situations.

Participatory Design Study

We began by conducting ten one-hour semi-structured interviews with students currently enrolled in a probability class. We found that less proficient students:

- crave concrete examples and visualizations,
- attempt but often fail to create their own representations, due not only to their lack of domain expertise but also to the limitations of pen and paper: one cannot draw a probability tree with 100 leaves,
- jump too quickly to abstract representations, e.g., formulas and mathematical notations, a major barrier to conceptual understanding,
- experience anxiety and cognitive load when faced with mathematical notations, and
- do not know where to start, often asking the teaching assistant to effectively solve the problem for them.

We next created a mockup with cardboard letters representing the building blocks of combinatorial problems. Participants could address questions such as: How many possible combinations of A, B and C are there? We also provided cardboard constraints to address questions such as: How many combinations obtain if A and B must be next to each other? Participants formed questions by combining physical letters and we created a corresponding visual representation (Fig. 2) with paper or on a whiteboard. One student suggested an innovative visualization, a kind of fractal representation that we tried with other students (Fig. 2, bottom). Based on these explorations, we designed Combinatorix, a custom-made tabletop with tangible objects that students manipulate to express and explore combinatorial problems.

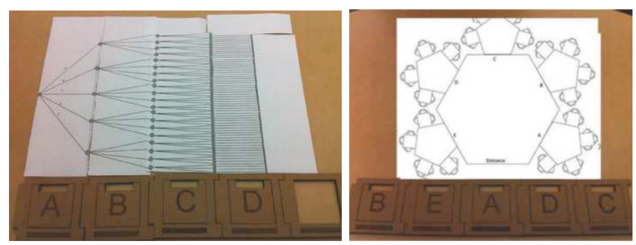


Figure 2. Participatory design: Cardboard mockup with paper-based tree (left) and graph representations (right)

COMBINATORIX

Hardware

Combinatorix (Fig. 4) supports several input techniques: a camera detects the location of fiducial markers and a wiimote provides the position of multiple infra-red pens. A projector displays additional information around the tangible objects. The interactive surface is 60 x 45 cm. and can accommodate up to four students at the same time.

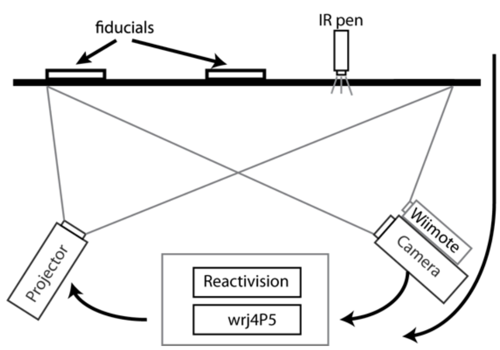


Figure 3. Combinatorix setup: The webcam detects location of fiducial markers; the wiimote detects position of infra-red pens

Software

The underlying application is written in Java and uses the Reactivision engine to detect fiducial makers [7]. Additional libraries, e.g., wrj4P50, communicate with the wiimote. The system is modular and can easily accommodate the creation of additional operators for constraining the sample space.

The current version displays two kinds of information: first, the tabletop interface shows a specific number of placeholders for objects. Letters can be placed on those spots to form a new combination. At the same time, the remaining number of letters for each step is displayed on top of each placeholder. A second screen displays a probability tree reflecting the current state of the problem. Letters can easily be replaced by other elements, including virtual, laser-cut and 3D-printed physical objects. Combinatorix supports up to 10 tangible objects and 20 virtual ones.

Interaction Techniques

Students can interact in two ways: 1) Use tangible letters to form combinations or to add constraints, e.g., fixing the position of a particular element. For example, Fig. 4 (top) shows the number of combinations when A and B are attached to each other. 2) Use a pen to annotate the probability tree. For example, Fig 4 (bottom) shows how to “prune” certain sections, which is equivalent to dividing a factorial number with the combinations that don't satisfy the constraint.

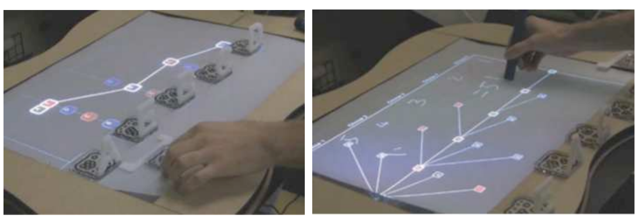


Figure 4. Users can switch between building combinations using physical objects (left) and annotating a probability tree (right).

Rational for Using Tangibles

We built on the results of Schneider, Jermann, Zufferey and Dillenbourg [8] to design the tangible part of our system. They found that compared to a multi-touch surface, a TUI better supported collaborative learning, users’ exploration of a problem space, playful learning and problem-solving strategies when working on a logistic problem. Our contribution is to go beyond those results and explore how the mapping of a specific concept to a tangible affects 1) problem-solving skills, and 2) transfer of those concepts to a new situation (that does not share the superficial features of a “school-type” problem). Ultimately, our goal is to provide students with a physical “toolbox” that encompasses all conceptual tools they can use to solve a probability problem. Table 1 summarizes the concepts that our system currently supports.

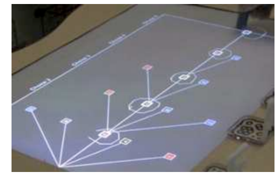
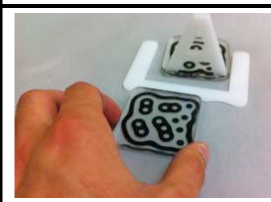
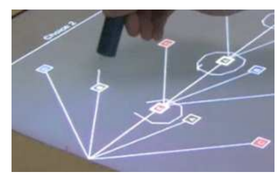
Concepts	Example	Tangible / action
1.Factorials *(multiplying the branches of the tree)*	“How many different linear arrangements are there of the letters A, B, C, D, E?”	
2.Grouping *(connecting two objects to group them)*	“ …for which A must be next to B?”	
3.Symmetry *(using a treemap to divide the sample space)*	“ …for which A is before B?”	
4.Position *(removing an object from the linear arrangement)*	“ …for which A is last in line?”	
5.Subtraction *(removing a subpart of the sample space)* **6.Permutation** *(pruning the tree)*	“ …for which A is not last in line?” “… with 4 placeholders?”	

Table 1. Conceptual mapping to physical objects and actions.

INFORMAL EVALUATION

Five participants tested Combinatorix, including two high-school students and three university students. We asked them to use the table to solve five problems of increasing difficulty: “The letters A, B, C, D, E form how many different linear arrangements?” 1) in total, 2) for which A and B are next to each other, 3) where E is not last in line, 4) for which A is before B, and 5) where A and B are next to each other and C is not first in line?".

General reception

Users were enthusiastic about using the system to solve the problems and were generally able to come up with the right solution after a few minutes. Problem four was the most difficult since the system does not provide any relevant hints. Instead, students tried a brute force solution, exhaustively counting the number of possible cases. The university students eventually realized that the problem was about symmetry: there is an equal number of combinations in which A is before B and B is before A. The solution is thus to divide the total number of combinations by two, e.g.

5! / 2. High school students required more support, in the form of prompts from the experimenter, to find this solution. Such prompts could easily be integrated into the system as automatic feedback; for instance, if a student spends too much time on a particular problem, Combinatorix could display a small hint to unblock the situation.

Participants found the current prototype very useful for certain types of problems: Ann[1] noted that "All the functionalities you could add should not do the thinking for the student; if I use this piece, it's telling what the solution is... well not really. It's more like a hint". Interviewer: "So do you think it's too much help?"; Ann: "I think it's a good level of help, because it conveys the notion that in this situation there are only four combinations that can be here".

However, Combinatorix clearly does not support all types of combinatorics problems. Henry said that "this is a really elegant way to show the concept of factorials; but for some problems I feel like I need to already know that concept to figure it out to get the solution". He also noted "it would be excessive to build a new model for each problem". This is the main challenge for our approach: some classes of problems can be supported easily, but others might require a totally different interface.

Although Combinatorix currently supports high-school level problems, future versions will address college-level problems including conditional probabilities (Bayes' theorem), independence of events, statistical indices (expected value, variance, standard deviation), discrete distributions (binomial, multinomial, geometric, hypergeometric, negative binomial), continuous distributions (uniform, normal, exponential, beta), law of large numbers and central limit theorem. We plan to support specific problems, such as the ones described, rather than creating a fully open-ended system, providing additional scaffolding to extend basic functions.

CONCLUSION

Due to the complexity of the domain i.e. combinatorics, and more generally probability, we do not envision Combinatorix as a stand-alone teaching tool. Rather, we consider it as a platform for students to reflect on problems, offload the cognitive burden of picturing all possible options and as a tool to provide small hints when students are stuck on a problem. Initial user testing revealed that students thought of it as a useful tool, but also mentioned important challenges that need to be addressed.

Our contribution is to develop a direct mapping between tangibles and concepts in combinatorics. To our knowledge, no previous work has studied the direct association of a concept to an arbitrary object in a learning environment. In the long run, our goal is to let students build their own objects and bring them home with them. They could then use those objects (that have been imprinted with a specific concept) to help them think about probability problems or transfer their knowledge in a different domain. These tangibles could be used as "reminders" of students' conceptual toolbox and thus be an ideal scaffold for transfer problems. For future studies and versions of Combinatorix, we intend to further develop its potential as a collaborative tool in a formal learning setting.

[1] All participant names have been anonymized.

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REFERENCES

1. Batanero, C. and Sanchez, E. What is the Nature of High School Students' Conceptions and Misconceptions About Probability? In G.A. Jones, ed., *Exploring Probability in School*. Springer US, 2005, 241–266.
2. Dillenbourg, P. and Evans, M. Interactive tabletops in education. *International Journal of Computer-Supported Collaborative Learning 6*, 4 (2011), 491–514.
3. Fast, G.R. Using analogies to overcome student teachers' probability misconceptions. *The Journal of Mathematical Behavior 16*, 4 (1997), 325–344.
4. Garfield, J. and Ahlgren, A. Difficulties in Learning Basic Concepts in Probability and Statistics: Implications for Research. *Journal for Research in Mathematics Education 19*, 1 (1988), 44–63.
5. Goldstone, R.L. and Son, J.Y. The Transfer of Scientific Principles Using Concrete and Idealized Simulations. *Journal of the Learning Sciences 14*, 1 (2005), 69–110.
6. Jendraszek, P.A. Misconceptions of probability among future teachers of mathematics. *Dissertation Abstracts International-A, 69*(01), 2008.
7. Kaltenbrunner, M. and Bencina, R. reacTIVision: a computer-vision framework for table-based tangible interaction. *Proceedings of the 1st international conference on Tangible and embedded interaction*, ACM (2007), 69–74.
8. Schneider, B., Jermann, P., Zufferey, G., et Dillenbourg, P. Benefits of a tangible interface for collaborative learning and interaction. *IEEE Transactions on Learning Technologies, 99*, 1–1
9. Schwartz, D.L. and Martin, T. Inventing to Prepare for Future Learning: The Hidden Efficiency of Encouraging Original Student Production in Statistics Instruction. *Cognition and Instruction 22*, 2 (2004), 129–184.
10. Steele, C.M. A threat in the air: How stereotypes shape intellectual identity and performance. *American Psychologist 52*, 6 (1997), 613–629.
11. Tversky, A. and Kahneman, D. Judgment Under Uncertainty: Heuristics and Biases. *Science* 185 (4157): 1124–11.