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Plausible deductive reasoning using a water tank analogy

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Abstract

The diagrammatic representation by "square-and-strips" used to illustrate and solve problems linked to conditional probability distributions is reinterpreted exclusively in terms of liquids in water tanks and applied to a new domain, namely deductive inferences under uncertainty. The isomorphism between the physical constraints on the capacity of the tank and its subdivisions and elementary laws of conditional probability is shown. Then, the analogy is exploited to qualitatively determine, and quantitatively compute, the limits in probability of the conclusion of basic deductive arguments (such as Modus Ponens, the Hypothetical Syllogism, Contraposition, etc.) that are used as benchmark problems by the various theoretical approaches to reasoning under uncertainty. It is shown that the probability bounds imposed by the premises on the conclusion and their respect by reasoners, stem from, and amount to, respecting trivial principles such as "a part of the tank cannot contain more than the whole tank's capacity", or "if a part is empty, the other part contains all the liquid", etc., suggesting a physical counterpart of Dutch book arguments to assess individuals' rationality.

Introduction

Many problems that imply the use of conditional probability are notoriously difficult. This fact is at the origin of the "Heuristic and Biases" school and the psychological debate about the format of representation of probability. It is also well known that some diagrammatic representations enhance performance on the solution of probability and statistical problems. Various representations have been proposed: Euler diagrams, line (or space) diagrams (Cheng, 2011), roulette wheel diagrams (Ichikawa, 1989), and square-and-strips diagrams, the earliest use of which is attributed to Edwards (1972) by Olford and Cherry (2006). Oldford (2003a, 2003b) called them "eikosograms". He shows how effective they are as an aid to represent the information and solve problems and puzzles as varied as the Engineer-and-Lawyer problem, medical tests, the prisoner's dilemma, the Monty Hall, Simpson's paradox, and many others. He alludes a few times to an interpretation of the diagrams in terms of a "water container metaphor" but does not pursue this idea. Furthermore, the scope of the papers is strictly limited to problem solving.

The present article is devoted to the development of an analogy of the laws of conditional probability applied to deductive schemas of inference under uncertainty, using a two-compartment water tank presented in a diagrammatic form. In the first section of the paper the basic diagram, the probabilistic interpretation of its components, and the representation of elementary laws of conditional probability are presented. In the second section a number of deductive inferences are interpreted in terms of the analogy, and the conditions under which it is rational to accept their conclusion are systematically determined. The last section briefly discusses the analogy from the viewpoint of its significance for the interpretation of the meaning of

probabilistic inferences, its usefulness for the evaluation of their limits, and its possible exploitation as an operational justification of Bayesian probability theory.

1. The water tank analogy

1.1. The basic diagram

A cubic tank has a movable partition that divides it into two compartments, left (A) and right (A'). Each compartment can contain some amount of liquid independently of the other (see Figure 1).

The dimensions of the tank are: height = 1 unit, width = 1 unit. The width of A is noted a ($0 \leq a \leq 1$).

The tank has a full capacity of 1 unit. Its overall content (filling rate) is noted c ($0 \leq c \leq 1$).

The levels of liquid in A and A' are noted i and i' , respectively. The amounts of liquid in A and A' are noted $[ac]$ and $[a'c]$. Similarly the empty spaces in A and A' are noted $[ac']$ and $[a'c']$.

The contents of A and A' are $i \times a$ and $i' \times a'$ (with $a' = 1 - a$), respectively.

$i \times a$ is a measure of $[ac]$, and $i' \times a'$ of $[a'c]$.

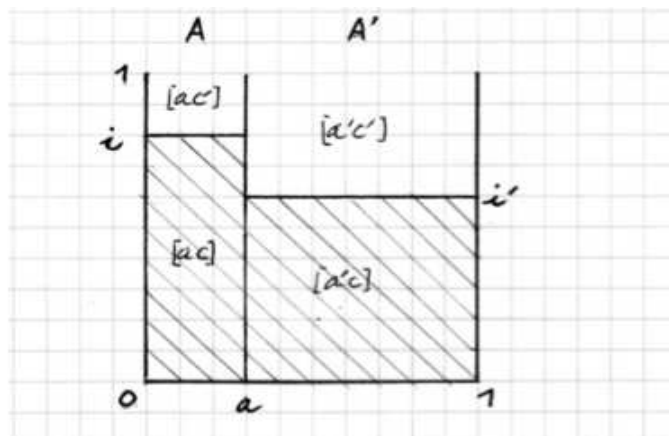


Figure 1. The basic analogy.

1.2. Probabilistic interpretation

$P(A) = a$ is the capacity of A ; $P(A') = a'$ is the capacity of A'.

$P(A \& C) = [ac]$, etc.

$P(C/A) = i$, so that the levels i and i' visually represent the two conditionals, $i = [ac] / a$ and $i' = [a'c] / a'$.

C is the event "the tank is filled with liquid C", $c = P(C/A)$ is the probability of this event, that is, the fraction of the tank that is filled with C.

A is the event "the left compartment occupies the whole tank", a the probability of this event, that is, the fraction of the whole tank that is occupied by A.

C/A is the event "the compartment A is full with liquid C", $i = P(C/A)$ the probability of this event, that is, the fraction of A that is filled. The tank is viewed as restricted to A.

A/C is the event "all the liquid C is in A", P(A/C) the probability of this event, that is, the fraction of liquid that is in A. The tank is viewed as restricted to the volume of the liquid.

As indicated by Oldford (2003a; 2003b; Oldford & Cherry, 2006) eikosograms allow the illustration of the total probability theorem, of Bayes's rule, and of probabilistic independence. With our current interpretation and notations, these are as follows.

1.3. Elementary rules

The total probability theorem

It is obtained by adding the contribution of each compartment to the whole.

$$c = [ac] + [a'c] = (a \times i) + (a' \times i')$$

Bayes' rule

The amount of liquid in A, [ac], can be viewed as the part of A that is filled, $a \times P(C/A)$, or as the part of c that is in A, $c \times P(A/C)$ (see Figure 2), hence:

$$a \times P(C/A) = c \times P(A/C), \text{ or: } P(A/C) = (a \times i) / c$$

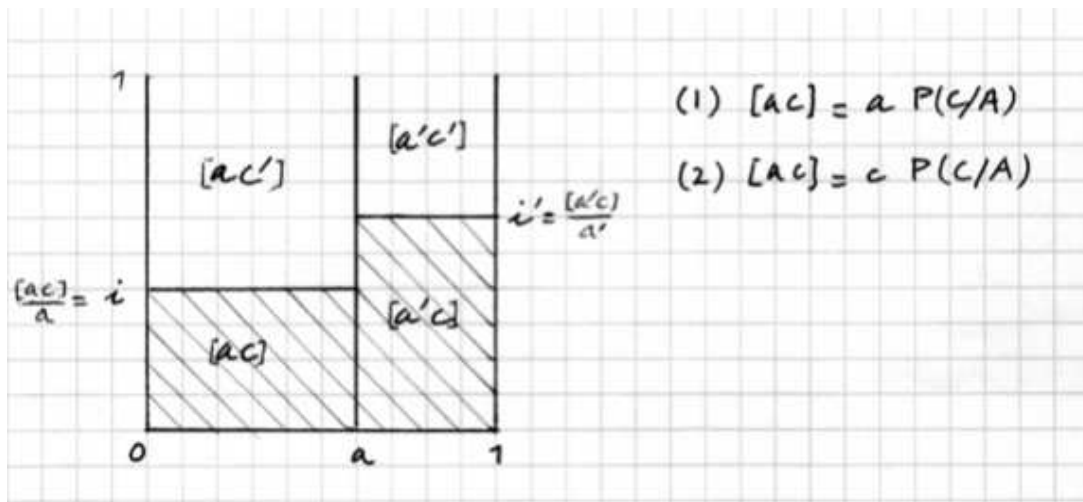


Figure 2. Bayes' rule.

Representation of independence

The levels are the same in A and A': $c = i = i'$, that is: $c = [ac] / a = [a'c] / a'$ (see Figure 3). In particular, $a \times c = [ac]$

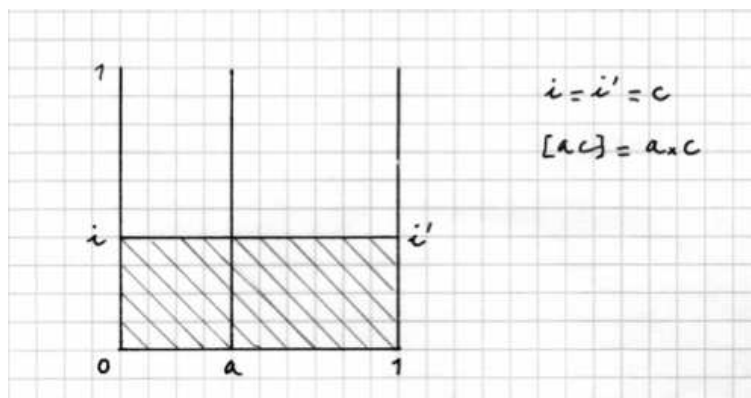


Figure 3. Independence.

2. Deduction under uncertainty

The recent interest in reasoning under uncertainty, more specifically in deductive reasoning with uncertain premises, has led to the elaboration of probabilistic logics (Adams, 1998; Coletti & Scozzafava, 2002; Gilio, 2002; Gilio & Over, 2012; Hailperin, 1996; Pfeifer & Kleiter, 2006, 2009; Politzer & Bourmaud, 2002). It is to this field of investigation that we are going to apply the tank analogy. The question that we are going to address is "What constraints do the probability of the premises impose on the probability of the conclusion?" And we will consider a related question for one-premise inferences: "What are the conditions for the probability of the conclusion to be no lower than that of the premise"? When it is no lower we will call the inference "conservative", and "dissipative" when it is no higher.

A major choice for probabilistic logics is to define a notion of validity that is the counterpart of logical validity in standard deductive logic. It is not the aim of the present paper to do so. Here the considerations are situated at a metatheoretic level. After the bounds in probability determined by Bayesian theory have been computed (which the tank analogy can do) theorists who have opted for a specific definition of validity can identify on a case by case basis which inferences are valid or invalid in their system.

Before considering inference arguments proper we show, to exemplify the method, how the bounds of a variable (e.g., the level i in one compartment) are constrained by the values of other variables (e.g., the capacity of one compartment and the whole content of the tank c) in a non-inferential situation.

2.1. The constraints of $a = P(A)$ and $c = P(C)$ on $i = P(\text{if } A \text{ then } C)$

We wish to determine the probability of the conditional *if A then C* knowing the probability of its components. Throughout this paper we follow Adam's (1996) thesis which equates the probability of a conditional sentence to the conditional probability of its consequent given its antecedent, $P(C/A)$.

The question amounts to the following: Given A's width a and the amount c to be poured, what are 1) the highest, and 2) the lowest possible levels in A?

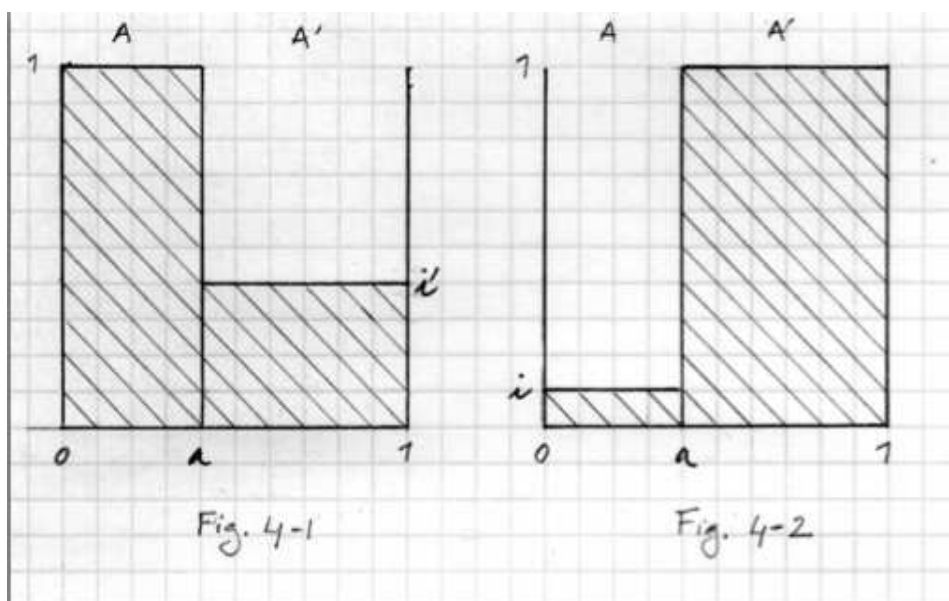


Figure 4. Constraint of $a = P(A)$ and $c = P(C)$ on $i = P(\text{if } A \text{ then } C)$.

1) One fills up A first. Trivially, $i = 1$ if $c \geq a$ (A is filled up when there is as much or more liquid than A can receive), and $i = a/c$ otherwise, hence: $i \leq \min \{c/a, 1\}$ (see Figure 4 (1)).

2) One fills up A' first. If $c \leq a'$, A remains empty and $i = 0$. If $c > a'$, A' is filled up and the excess is poured into A which receives $c - a'$, hence its level $(c - a') / a$ or equivalently $(c - 1 + a) / a$. Hence: $i \geq \max \{0, (c - 1 + a) / a\}$ (see Figure 4(2)). Therefore:

$$\max \{0, (c - 1 + a) / a\} \leq i \leq \min \{c/a, 1\}$$

We now review in turn a number of inference schemas.

2.2. And-introduction: $A ; C \therefore A \& C$

Given \underline{a} and the amount \underline{c} to pour, trivially the part of \underline{c} in A, $[ac]$, cannot exceed \underline{a} nor can it exceed \underline{c} : $[ac] \leq \min \{a, c\}$

Filling A', if $c \leq 1 - a$, A remains empty and $[ac] = 0$; if $c > 1 - a$, A receives $c - (1 - a)$, hence: $[ac] \geq \max \{a + c - 1, 0\}$.

$$\max \{a + c - 1, 0\} \leq [ac] \leq \min \{a, c\}$$

$$[ac] \in [\max \{a + c - 1, 0\}, \min \{a, c\}]$$

Breaking the upper bound amounts to committing the *conjunction fallacy* by which the conjunction of two events is estimated as more likely than one of the conjuncts.

**2.3. And-elimination: $A \& C \therefore A$
 $A \& C \therefore C$**

There is always increase in probability from premise to conclusion: the amount of liquid in A, $[ac]$, being fixed, it is trivially smaller or equal to either the capacity of A, \underline{a} , or the total amount of liquid \underline{c} . Or equivalently, if A contains $[ac]$, it cannot contain less (but it can contain more); and similarly if some of C is in A there cannot be less liquid overall but there can be some more in A'. The inference is **always conservative**.

$$[ac] \leq a; \quad [ac] \leq c.$$

Calling α the premise and ω the conclusion: $P(\alpha) \leq P(\omega) \leq 1$

2.4. And to if: $A \& C \therefore A \rightarrow C$

Imagine $[ac]$ fixed and the partition fixed at the extreme right ($a = 1$), the level being i (Figure 5-1). Now moving the partition leftwards (keeping A' empty), i can only increase (Figure 5-2), showing that we always have $i \geq [ac]$. The limit is reached when $[ac] = a$ and $i = 1$ (Figure 5-3). Arithmetically of course, $i \geq [ac]$ because the ratio $i = [ac] / a$ is ≥ 1). In other words, if you can warrant an amount of liquid at least as great as $[ac]$ in A, you also warrant a level in A that is at least as high as $i = [ac] / a$. This is the meaning of the inference from *and* to *if*, showing why it is **conservative**:

$$P(\alpha) \leq P(\omega) \leq 1.$$

For the same reason, the reverse inference from *if* to *and* is always dissipative.

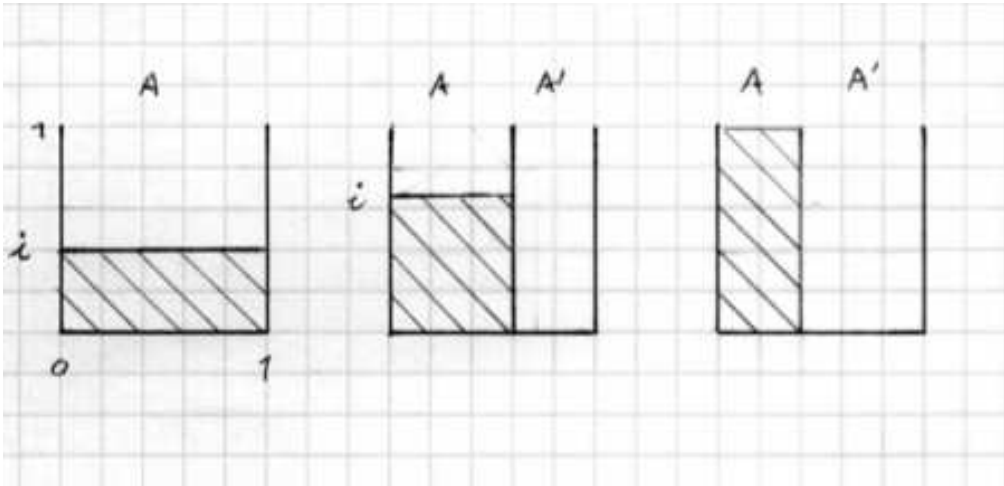


Figure 5. Inference from AND to IF.

2.5. And to or: $A \& C \therefore A \vee C$

The content of A is [ac]: it cannot exceed the capacity of A; therefore, a fortiori, it cannot exceed the capacity of A augmented with the part of C, [a'c], that A' contains (see Figure 6). The inference is **always conservative**.

$$P(\alpha) \leq P(\omega) \leq 1$$

Arithmetically: $[ac] \leq [ac] + [a'c]$

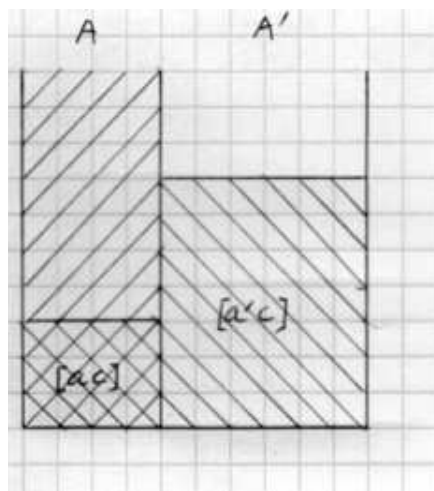


Figure 6. Inference from AND to OR.

2.6. Or-introduction: $A \therefore A \vee C$

This is a particular case of *and to or* with A full ($[ac] = a$). It is therefore **always conservative**.

$$P(\alpha) \leq P(\omega) \leq 1$$

2.7. Or to if: $A \vee C \therefore \text{not-}A \rightarrow C$

This inference can be treated in two ways (see Figure 7).

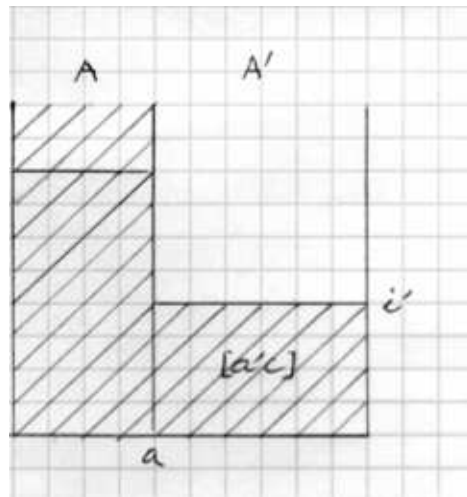


Figure 7. Inference from OR to IF.

Firstly, the conclusion represents the level in A', while the premise represents the capacity of A augmented with the part of the liquid that is in A'. Now trivially, the global rate calculated on the basis of A being full necessarily exceeds (or equals) the rate of A' only, showing that one cannot warrant a level in A' that would numerically exceed the content of A' augmented with the capacity of A. This is why the probability of the conclusion can never exceed that of the premise: the inference is **dissipative**.

Secondly, consider the probability of the premise $A \vee C$ fixed (noted d):

$$[a'c] + a = d$$

$$(a' \times i') + a = d$$

$$i' = (d - a) / 1 - a$$

The graph of i' as a function of d with a as a parameter (Figure 8) shows that i' is a linearly increasing function of d and that we always have $i' \leq d$, showing that the inference is non-increasing (except trivially when $a = 0$ and then $i' = d = 1$) and that the limits for i' are: $0 \leq i' \leq d$, hence: $0 \leq P(\omega) \leq P(\alpha)$.

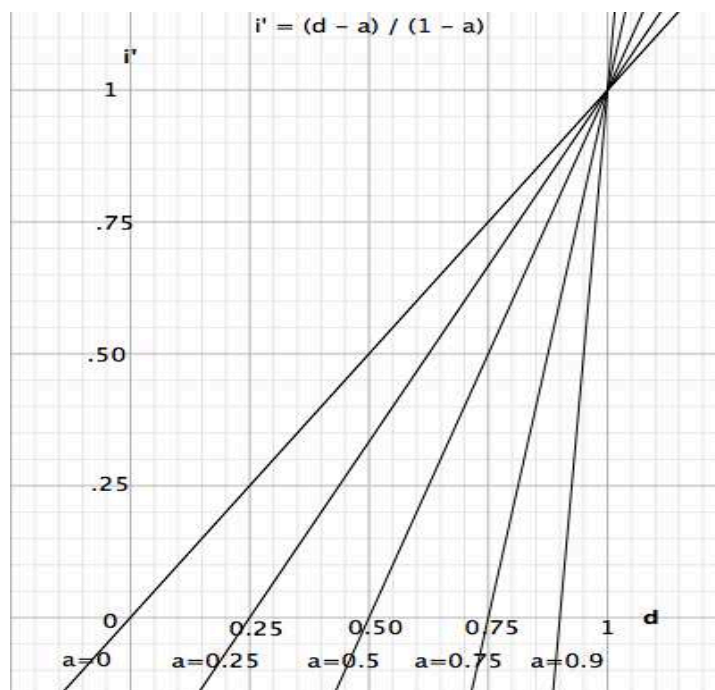


Figure 8. Inference from OR to IF. Variation of $i' = P(\text{not } A \rightarrow C)$ as a function of $d = P(A \text{ or } C)$ for various values of $a = P(A)$.

2.8. Contraposition: $A \rightarrow C \therefore \text{not-}C \rightarrow \text{not-}A$

The conclusion represents the extent to which the right compartment contributes to the vacuum (the emptiness of the tank). This rate will be noted i_c .

Trivially when the left compartment is full ($i = 1$, full belief in the premise) i_c equals 1, in which case contraposition is conservative (see Figure 9-1).

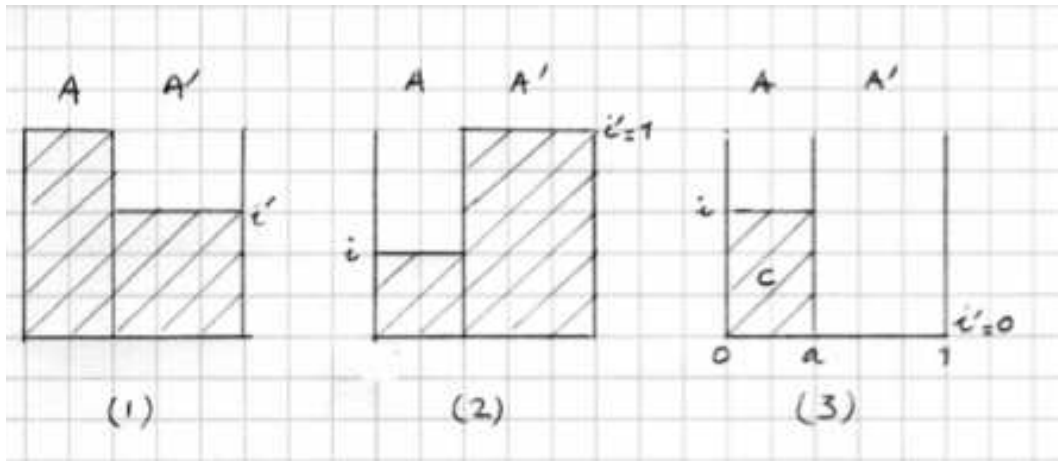


Figure 9. Contraposition.

Also if A occupies the whole tank ($a = 1$), the empty volume is entirely in A and $i_c = 0$, the inference is dissipative.

When the right compartment is full ($i' = 1$) its contribution to the vacuum is null, $i_c = 0$, the inference is dissipative (Figure 9-2). As the level in A' decreases, i_c increases, reaching a maximum when A' is empty (Figure 9-3) equal to $(1 - a) / (1 - c)$. Depending on the value of a , this may be greater or smaller than i .

Algebraically: $i_c = (1 - a)(1 - i') / (1 - c)$

$$i_c = (1 - a)(1 - i') / [(1 - a)(1 - i') + a(1 - i)] \quad (\text{Eq})$$

This is the fundamental equation for contraposition that we rewrite as:

$$y = (1 - a)(1 - i') / [(1 - a)(1 - i') + a(1 - x)] \quad (\text{Eq})$$

expressing the functional relation between the contrapositive $y = i_c$ and the conditional $x = i$ with a and i' as parameters. We rewrite (Eq) as

$$y = u / (v - ax), \text{ with } u = (1 - a)(1 - i'), \text{ and } v = u + a.$$

It can be verified that this function is strictly increasing (first derivative strictly positive) and that it is concave (second derivative strictly positive).

Knowing whether the inference is conservative or not amounts to knowing the conditions under which $y \geq x$, that is: $u / (v - ax) \geq x$, or:

$$ax^2 - vx + u \geq 0 \quad (\text{InEq})$$

The associated equation can be factorised as:

$$a(x-1)(x - u/a) = a(x-1)(x-j) \text{ where } j = u/a = (1-a)(1-i')/a.$$

As a result, the sign of the difference $y-x$ varies as a function of a as follows:

- if $j \geq 1$ there is no solution (other than $x=1$) within $[0, 1]$, and (InEq) is always satisfied, which means:

for all $i \in [0, 1]$, $i_c \geq i$, that is the inference is **conservative** whenever:

$$j = (1-a)(1-i')/a \geq 1, \text{ that is, whenever } i' \leq (1-2a) / (1-a) \text{ or equivalently } a \leq (1-i') / (2-i')$$

- if $j < 1$ there is a solution smaller than 1 which means that (InEq) is satisfied on an interval $[0, j]$ and not satisfied on $[j, 1]$. This means that whenever

$i' > (1-2a) / (1-a)$ or equivalently $a > (1-i') / (2-i')$ we have:

$$i_c > i : \text{ a conservative inference for } i \in [0, (1-a)(1-i')/a],$$

and $i_c < i : \text{ a dissipative inference for } i \in [(1-a)(1-i')/a, 1].$

Figure 10 illustrates this discussion for the value of the parameter i' fixed at $i' = .4$ and for several values of the parameter a ranging from 0.2 to 0.95.

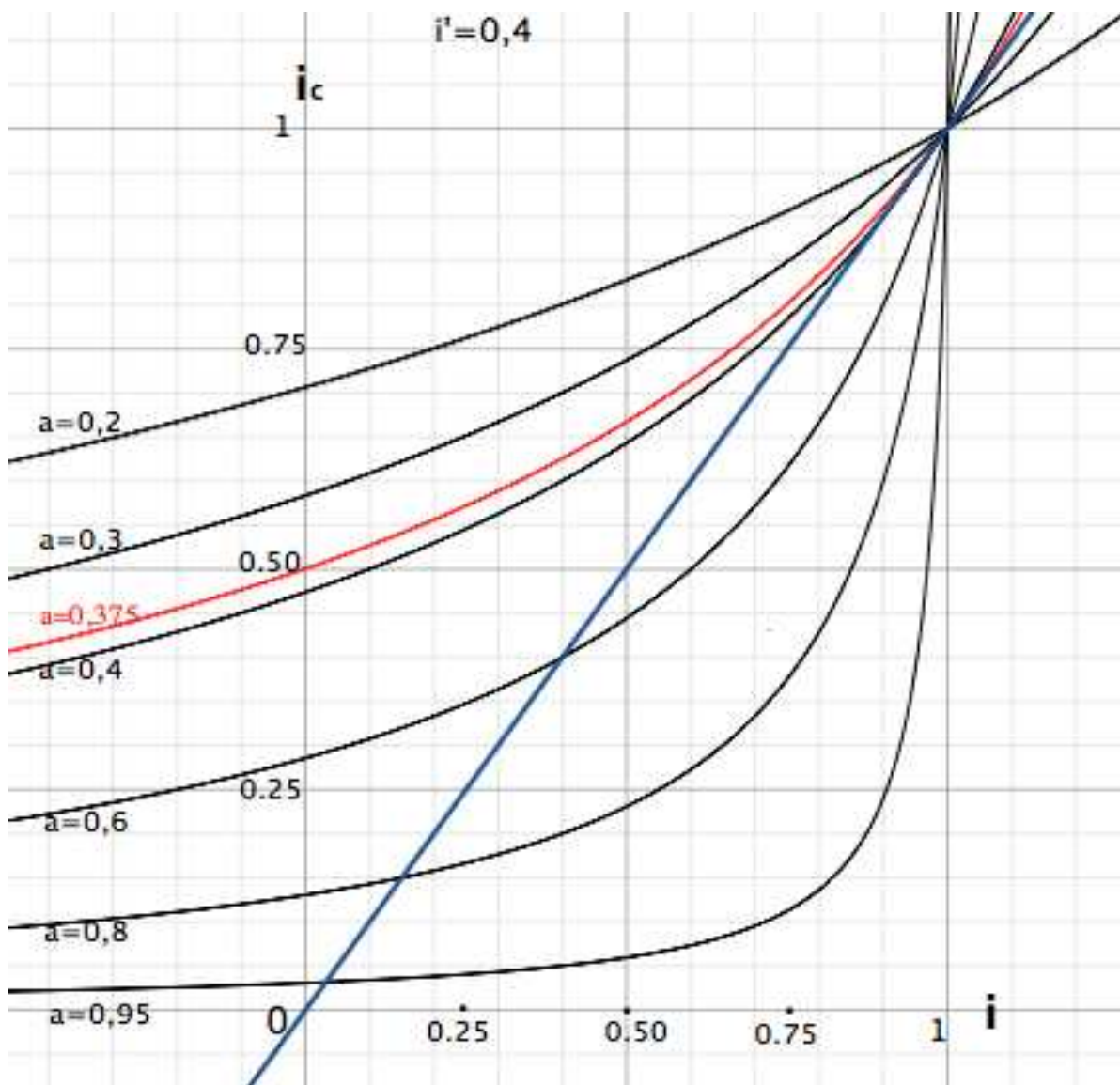


Figure 10. Domain of validity of contraposition for $i' = P(C/\text{not } A) = 0.4$ and various values of $i = P(C/A)$. Contraposition is conservative for $0 \leq i \leq j$ ($j =$ intersect of $f(i)$ and the diagonal). Case $a \leq (1-i') / (2-i')$.

2.9. Modus Ponens $A \rightarrow C; A \therefore C$

The combination of the premises \underline{a} and \underline{j} yields $a \times i = [ac]$ as the content of A. Obviously the content of the whole tank, \underline{c} cannot be less, which gives a lower bound for the confidence in C: one must not be less confident in the conclusion than in the product of the levels of confidence in the premises. On the other hand, A being limited to level \underline{j} , the tank cannot be full; \underline{c} cannot exceed the content of A, $a \times i$, augmented with the whole capacity of A', that is $a \times i + (1 - a)$ (see figure 11). Therefore:

$$a \times i \leq c \leq (a \times i) + 1 - a$$

$$c \in [a \times i, (a \times i) + 1 - a]$$

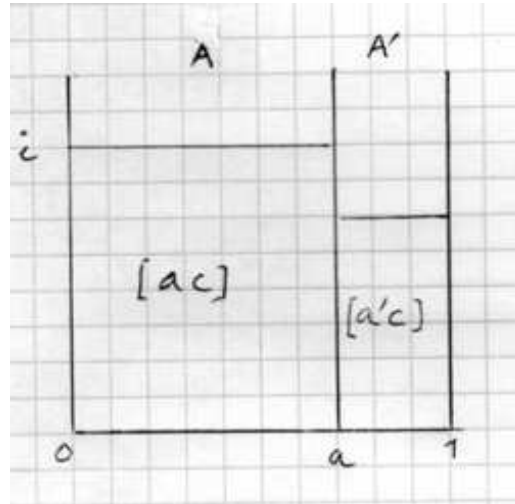


Figure 11. Modus Ponens.

Notice that when A occupies the whole tank, the level of the tank is given by \underline{j} : $c = i$ and \underline{c} is known with the least uncertainty (as equal to \underline{j}). Now suppose A decreases while \underline{j} is fixed (say = 1). A' increases and because \underline{j} is unknown the uncertainty on \underline{c} increases; when \underline{a} becomes null A' occupies the whole tank and \underline{c} can be anything between 0 and 1: this is the non probabilistic case of Denying the Antecedent where $i=1$ and $a=0$. This shows that MP and DA are the two sides of a same coin; the variable is A in one case and A' in the other case.

2.10. Denying the Antecedent $A \rightarrow C; C \therefore A$

The same equation as for MP can be used, replacing \underline{c} with $c' = 1 - c$, which yields:

$$a - (a \times i) \leq c' \leq 1 - (a \times i)$$

$$c' \in [a - (a \times i), 1 - (a \times i)]$$

This represents the degree of emptiness of the tank knowing the level in A and the size of A'. The equation shows that it is rational to have a level of confidence in the conclusion of DA provided it lies between the bounds indicated. In particular, trivially, it would be irrational to have a confidence in the total void lower than $a - (a \times i)$, that is, lower than the void in A.

The result can also be given in terms of a':

$$c' \in [(1 - i)(1 - a'), 1 - i(1 - a')]$$

2.11. Affirming the Consequent $A \rightarrow C; \quad C \therefore A$

Given the level in A and the content \underline{c} , trivially there cannot be more liquid in A than the whole amount \underline{c} , that is: $a \times i \leq c$, hence: $a \leq c / i$ (which is automatically satisfied when $i \leq c$ since $a \leq 1$).

Similarly, A cannot incorporate more void than the whole void in the tank, that is:

$(1 - i) \times a \leq 1 - c$, hence $a \leq (1 - c) / (1 - i)$ (which is automatically satisfied when $c \leq i$ since $a \leq 1$). It follows that:

$$a \in [0, c/i] \text{ if } i \geq c$$

$$a \in [0, (1 - c)/(1 - i)] \text{ if } i \leq c$$

Figure 12 represents the graph of \underline{a} plotted against \underline{c} for two values of \underline{i} as a parameter. For a given value of \underline{i} , the range of \underline{a} is obtained by referring to the triangle whose base is $[0, 1]$ and the apex on the line $a=1$. For any value of \underline{c} , the ordinate of the corresponding side of the triangle provides the range of \underline{a} . It is noteworthy that when \underline{c} nears 1 \underline{a} becomes very small, and null when $c=1$. (The exception to this occurs when $i=1$ and $c=1$ in which case $a \in [0, 1]$, which is the non probabilistic fallacy of AC). If you know that the left part is full to three quarters but also that the tank is nearly full, the only possibility is that the left part be very small.

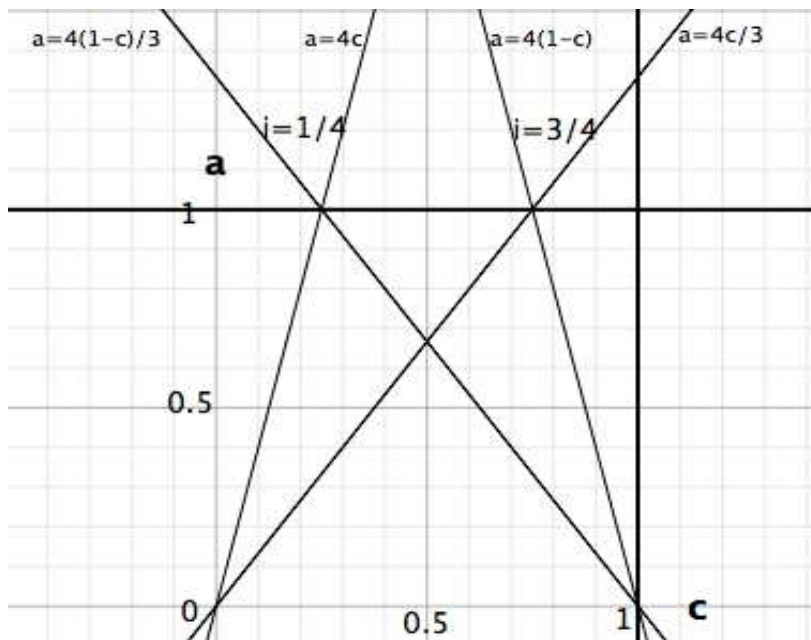


Figure 12. Inference of Affirming the consequent. Graph of the conclusion $a = P(A)$ as a function of the minor premise $c = P(C)$ for two values of the major premise $i = P(A \rightarrow C)$.

Suppose now that one has some belief in \underline{i} ($i > .5$) and disbelieves \underline{c} ($c < .5$): the range of \underline{a} is measured by the ordinate on the left side of the triangle: the more \underline{c} decreases, the more \underline{a} decreases. This is the probabilistic form of Modus Tollens, showing its link with AC. In particular when $i=1$ \underline{a} must be null: this is the non probabilistic MT. The solution of MT follows.

2.12. Modus Tollens $A \rightarrow C; \text{ not-C} \therefore \text{ not-A}$

Knowing the level in compartment A and the size of the vacuum in the tank, what are the limits of the right compartment?

We introduce $c' = 1 - c$ and $a' = 1 - a$ in the result for the Affirmation of the Consequent, hence the solution:

$$a' \in [(i - 1 + c') / i, 1] \quad \text{if } i \geq 1 - c'$$

$$a' \in [(1 - i - c') / (1 - i), 1] \quad \text{if } i \leq 1 - c'$$

2.13. Hypothetical syllogism: $A \rightarrow B; B \rightarrow C \therefore A \rightarrow C$

In this case the volume occupied by liquid B is partly replaced by liquid C. In the certain, classic, case compartment A is entirely filled with liquid B and then the whole of liquid B is replaced with liquid C, so that in the end A is filled with C and the inference is valid. In the probabilistic case, the problem is that even if the level in A is high ($P(B/A)$ high) and a large volume of B is replaced ($P(C/B)$ high), when some of the liquid B is replaced this could be done by removing it from A' while little or no liquid B is removed from A, making the level of C in A ($P(C/A)$ low or even null (Figure 13). In brief, no confidence value can be given for the conclusion: $0 \leq P(\omega) \leq 1$.

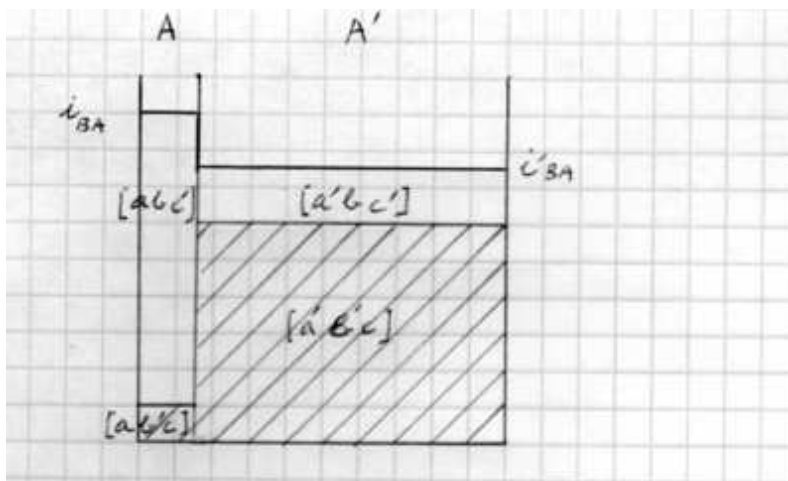


Figure 13. Hypothetical syllogism.

This is to be contrasted with CUT: $A \rightarrow B; A \& B \rightarrow C \therefore A \rightarrow C$

The second premise indicates that the replacement of B by C must be done by removing B from A so warranting that there will be some C in A and the probability of the conclusion cannot be null.

2.14. Strengthening: $A \rightarrow C \therefore (A \& B) \rightarrow C$

Compartment A is divided into two sub-compartments B and B'. In the classic case (Figure 14-1) A is assumed to be full so that any subpart of it such as B will be full as well, hence the validity of the inference.

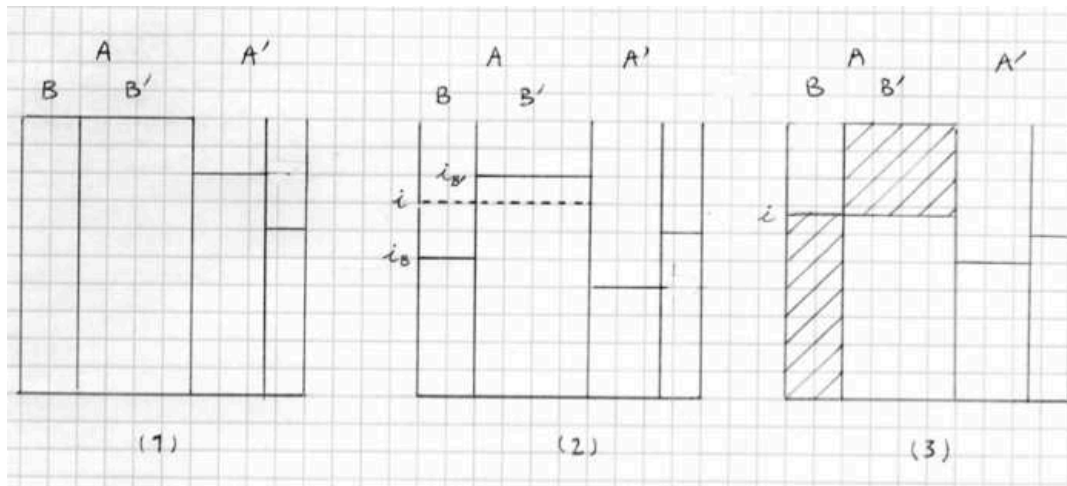


Figure 14. Strengthening of the antecedent.

In the probabilistic case there is no guarantee that A is full and the sub-compartments need not have the same level (Figure 14-2). $P(C/A) = i$ is an average value between $i_B = P(C/AB)$ and $i_{B'} = P(C/AB')$. Strengthening the antecedent with B can decrease the probability of the conditional (as strengthening with B' can increase it). In particular, the level in AB is null in case of a strict invalidating condition (or exception, or defeater).

An obvious condition for strengthening is that the levels i_B and $i_{B'}$ differ, that is, there is probabilistic dependence between A and B: $P(C/AB) \neq P(C/AB')$. It is always possible for i_B to exceed i strictly, except when $c = 0$ or $c = 1$.

The condition under which AB can be empty, $P(C/AB) = 0$, is that there be enough empty space in AB' to accommodate the liquid in AB (Figure 14-3, the hatched rectangles have equal areas). This gives $b \leq 1 - c$ as a necessary condition. If i is relatively high as is normally the case when a conditional is asserted, $i_B = 0$ can occur to the extent that b is small enough, which is exactly the sense of B being an exceptional event. This is the well-known case where a conditional assertable with antecedent A must be retracted when A is qualified by a defeater B. Similarly, i_B can reach 1 provided $b \leq c$.

More generally, assuming B to be the smaller compartment, its level i_B depends on the size of c following a relation already given:

$$\begin{aligned}
 i_B \in [0, 1] & & \text{if } b \leq c \leq 1 - b \\
 i_B \in [0, c/b] & & \text{if } c \leq b \\
 i_B \in [(c + b - 1)/b, 1] & & \text{if } c \geq 1 - b
 \end{aligned}$$

2.15. Right-nested conditionals and import-export: $(A \& B) \rightarrow C \therefore A \rightarrow (B \rightarrow C)$

A conditional is represented by a level and its consequent by a amount of liquid. In the conclusion of this inference ($B \rightarrow C$) is a *level* qua conditional but at the same time it must be an *amount* qua consequent of a conditional. This confusion prevents nested conditionals, and therefore this inference, to be represented in the system without some accommodation, which is beyond the scope of this paper.

3. Conclusion

The tank analogy is worth considering from at least two points of view, theoretical and practical.

The main interest of the tank analogy is that it provides an interpretation of the meaning of probabilistic deductive inferences. Atomic sentences are interpreted in terms of volumes or capacity of compartments, and conditionals in terms of levels. Boolean expressions have their interpretation in terms of union or intersection of volumes or compartments so that the logical operations have their physical counterparts, which makes the analogy an isomorphism. Varying the value of the constituents (volumes, capacities and levels) provides the probabilistic interpretation. Overall, an inference is viewed as the statement of the physical feasibility of the configuration described in the conclusion, given the configuration (the constituents) specified in the premise(s). The feasibility may be warranted (conservative case) or unwarranted (dissipative case) without conditions, or it may be warranted on the condition that some constraint on the relative values of the constituents in the premise be satisfied; this results in an upper and/or a lower bound for the value of the constituent's conclusion. So, we have an operational procedure to evaluate normatively, that is, from the viewpoint of probability theory, deductive inferences under uncertainty. It can be seen that the evaluation of the conclusion amounts to a search for compatibility or coherence between the configurations described by the premise(s) and the conclusion. It would be incoherent, if one accepts the premise configuration, to expect a conclusion configuration that is physically impossible. This offers an alternative to betting situations and Dutch book arguments classically applied to assess individuals' rationality in their probability judgments: in a similar way that individuals whose bets do not conform to coherent probability bounds are doomed to losing their money, the same individuals commit themselves to the execution of impossible fillings.

Practically, one advantage linked to the operational procedure of the analogy over formal methods of calculation is that it provides both a qualitative method (which opens didactic perspectives) and a basis for the quantitative calculation of the bounds.

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