Generation of the coupling circuit parameters for the coupled oscillators used in antenna arrays
Mihaela-Izabela Ionita, Mihai Iordache, Dumitriu Lucia, David Cordeau, Jean-Marie Paillot

To cite this version:

HAL Id: hal-00862712
https://hal.science/hal-00862712
Submitted on 17 Sep 2013

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers. L’archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d’enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.
Abstract—This paper presents a new software which can generate in full-symbolic or numeric-symbolic form the $Y$, $Z$, $H$, and fundamental parameters of a two-port structure. Our procedure can also determine all the resonant frequencies of any two-port configuration as functions of the two-port circuit parameters. The procedure is based on the modified nodal equations in full-symbolic form. A new software called ANCSYANP (Analog Circuit Symbolic Analysis Program) was elaborated. This is an interactive tool that combines symbolic and numeric computational techniques, and which uses the facilities of the symbolic simulator Maple to manipulate the symbolic expressions. An illustrative example is done.

Keywords-component; symbolic analysis; two-port circuit; coupled oscillator; $Y$, $Z$, $H$ parameters

I. INTRODUCTION

In the last years the coupled oscillators are used to control the phase in microwave antenna arrays. These devices produce oscillatory output signals of high frequency [1-9]. The radiation pattern of a phased antenna array is steered in a particular direction by controlling the phase gradient existing between the signals applied to adjacent elements of the array. The required inter-element phase shift can be obtained by detuning the free-running frequencies of the outermost oscillators in the array [2]. Furthermore, in [4] it is shown that the resulting inter-stage phase shift is independent of the number of oscillators in the array.

The aim of this paper is to present the symbolic analysis of the array of two coupled Van der Pol oscillators, considering the coupling circuit as a passive two-port circuit. Therefore, a new software which can generate in full-symbolic or numeric-symbolic form the $Y$, $Z$, $H$, and fundamental parameters of any two-port structure was developed. The procedure is based on the modified nodal equations of the entire circuit formulated in full-symbolic form.

The paper is organized as follows: a system of two Van der Pol oscillators coupled through a $RLC$ circuit is presented in section II. In section III, the symbolic generation of the coupling circuit parameters with the new software is described followed by conclusions.
The choice of this model is justified by its simplicity regarding the analytical calculations, as presented in [5, 6].

The active parts of the two Van der Pol oscillators are modeled by two voltage-controlled nonlinear resistors. The nonlinear characteristics of the two voltage-controlled nonlinear resistors are expressed as follows:

\[ i_{\text{net}1} = -av_1 + b v_1^3; \quad i_{\text{net}2} = -av_2 + b v_2^3, \]  

(1)

where \( a \) is the negative conductance necessary to start the oscillation and \( b \) a parameter used to model the saturation phenomenon.

Assuming a perfect oscillation so that \( v(t) = A \cos(\omega_0 t) \) and according to (1), the expressions of the currents through the two nonlinear resistors can be written as:

\[ i(t) = \left( -a + \frac{3}{4}bA^2 \right) A \cos(\omega_0 t) + b A^3 \cos(3\omega_0 t)/4 \]  

(2)

Therefore, the two Van der Pol oscillators in Fig. 1, b can be modeled by a quasi-linear representation, replacing the two nonlinear resistors by the following conductances:

\[ G_{02} = -a + \frac{3}{4}bA_1^2 \quad \text{and} \quad G_{09} = -a + \frac{3}{4}bA_2^2, \]  

(3)

where \( A_1 \) and \( A_2 \) are the magnitudes of voltages \( v_1 \) and \( v_2 \).

Performing a Spice simulation in transient behavior we get the magnitudes values \( A_1 = 1.6032 \, \text{V}, A_2 = 2.4431 \, \text{V}, \) and the synchronization frequency \( f_s = 1.1512 \, \text{GHz}, \) for the initial conditions \( v_1(0) = 2.0 \, \text{V} \) and \( v_2(0) = 1.0 \, \text{V}. \) Thus, in sinusoidal behavior the two voltage-controlled nonlinear resistors can be substituted by two linear resistors having negative slopes. In this case the circuit in Fig. 1 can be analyzed by the complex representation method [10, 11].

### III. SYMBOLIC GENERATION OF THE COUPLING CIRCUIT PARAMETERS

The aim of this section is to generate in full-symbolic or numeric-symbolic form the \( Y, Z, H, \) and fundamental parameters of the coupling circuit of the two Van der Pol oscillators. To this end the coupling circuit is represented by a passive linear two-port circuit, as shown in Fig. 2.

![Diagram of a general scheme of two coupled oscillators](Image)

In these conditions, the equations in complex admittances of the passive linear two-port are expressed as follows:

\[ \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \]  

(4)

where the transfer admittances are defined in the expressions below:

\[ Y_{11} \frac{d}{dt} \frac{v_1}{v_{12}} = Y_{12}; \quad Y_{12} \frac{d}{dt} \frac{v_1}{v_{21}} = -Y_{11}; \]  

\[ Y_{21} \frac{d}{dt} \frac{v_2}{v_{21}} = Y_{22}; \quad Y_{22} \frac{d}{dt} \frac{v_2}{v_{12}} = -Y_{21}. \]  

(5)

Furthermore, the equations in complex impedance of a passive linear two-port can be written as:

\[ \begin{bmatrix} Z_1 \\ Z_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \]  

(6)

where the transfer impedances are defined as follows:

\[ Z_{11} \frac{d}{dt} \frac{v_1}{v_{11}} = Z_{12}; \quad Z_{12} \frac{d}{dt} \frac{v_1}{v_{22}} = -Z_{11}; \]  

\[ Z_{21} \frac{d}{dt} \frac{v_2}{v_{21}} = Z_{22}; \quad Z_{22} \frac{d}{dt} \frac{v_2}{v_{11}} = -Z_{21}. \]  

(7)

In \( H \) parameters the equations are:

\[ \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \]  

(8)

and the definitions of the parameters are:

\[ H_{11} \frac{d}{dt} \frac{v_1}{v_{11}} = Z_{11}; \quad H_{12} \frac{d}{dt} \frac{v_1}{v_{22}} = -Z_{11}; \]  

\[ H_{21} \frac{d}{dt} \frac{v_2}{v_{21}} = Z_{21}; \quad H_{22} \frac{d}{dt} \frac{v_2}{v_{11}} = -Z_{21}. \]  

(9)

If the fundamental (transfer) complex parameters are of interest, the equations have the form:

\[ \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \]  

(10)

and the parameters are defined as:

\[ A = \frac{1}{v_{11}}; \quad B = \frac{1}{v_{22}}; \quad C = \frac{1}{v_{11}}; \quad D = \frac{1}{v_{22}}. \]  

(11)
In order to automatically generate (in symbolic or numeric-symbolic form) all the above parameters associated to the two-port used to model the coupling circuit, we adapted the general software - ANCSYANP (Analog Circuit Symbolic Analysis Program) [10, 11]. The new analysis tool based on the modified nodal analysis (MNA) generates, starting from the circuit netlist, the Y, Z, H, and fundamental parameters, for any linear and/or nonlinear (in any driving point) time-invariant two-port analog circuit, in symbolic form. The excess elements are also taken into account. It is an interactive tool that combines symbolic and numeric computational techniques, and which uses the facilities of symbolic simulator Maple to manipulate the symbolic expressions.

Let us now remember the equations in Laplace domain corresponding to the MNA:

\[
\begin{bmatrix}
Y_{n,n-1}(s) \\
B_{n,n+1}(s) \\
A_{n,n-1}(s) \\
Z_{n,n+1}(s)
\end{bmatrix}
\begin{bmatrix}
V_{n-1}(s) \\
V_{n+1}(s) \\
I_{n-1}(s) \\
I_{n+1}(s)
\end{bmatrix}
= \begin{bmatrix}
I_{n,n-1}(s) \\
E_{n,n+1}(s) \\
E_{n,n-1}(s) \\
E_{n,n+1}(s)
\end{bmatrix},
\]  
\hspace{1cm} (12)

where: \( Y_{n,n-1}(s) \) is the operational admittance matrix of size \((n-1)\times(n-1)\); \( B_{n,n+1}(s) \) represents a matrix of size \((n-1)\times m\) with entries -1, 0, 1 and the current transfer factor (gain) of the current-controlled current sources; \( A_{m,n-1}(s) \) is a \( m\times(n-1) \) matrix with entries -1, 0, 1 and the voltage transfer factor (gain) of the voltage-controlled voltage sources; \( Z_{m,n+1}(s) \) represents a square matrix of \( m\times m \) size which contains the operational trans-impedances of the current-controlled voltage sources and the operational inductor impedances with negative sign; \( I_{n,n-1}(s) \) is the operational short-circuit current vector injected in \( n-1 \) independent nodes (including the ones resulted from the independent sources which simulate the initial conditions) and \( E_{m,n+1}(s) \) represents the vector of the Laplace transforms (with negative sign) of the electromotive forces corresponding to the ideal independent voltage sources and of the electromotive forces corresponding to the ideal independent voltage sources which simulate the inductor initial conditions. The unknown vector contains the Laplace transform vector of the electrical potentials corresponding to the \( n-1 \) independent circuit nodes, \( V_{n-1}(s) \), and the Laplace transform vector of the branch currents non-compatible with the nodal analysis, \( I_{n}(s) \).

Based on the definition of the circuit parameters we apply suitable ideal independent sources to the input ports of the circuit in Fig. 2. Describing the circuit behavior by MNM the program computes in symbolic form the desired parameters.

ANCSYANP has the following capabilities:

- Generation in full symbolic, partially symbolic or numeric form of the two-port parameters;
- Computation of the sensitivities with respect to any circuit parameter and the 3D representation;

In general, \( Y \) parameters are important in the analysis of two oscillators coupled through a passive linear two-port circuit. For the circuit in Fig. 1, a, the equations in complex form, when the two voltage-controlled nonlinear resistors are substituted by two linear resistors according to (3), can be written as:

\[
\begin{align*}
Y_1 Y_1 &= -Y_{11} Y_1 - Y_{12} Y_2 \\
Y_2 Y_2 &= -Y_{21} Y_1 - Y_{22} Y_2,
\end{align*}
\]  
\hspace{1cm} (13)

where

\[
\begin{align*}
Y_1 &= -a + \frac{3}{4} b h A_1^2 + j o C_1 + \frac{1}{2} \frac{1}{R} \\
Y_2 &= -a + \frac{3}{4} b h A_2^2 + j o C_2 + \frac{1}{2} \frac{1}{R}
\end{align*}
\]  
\hspace{1cm} (14)

Running ANCSYANP all transfer admittances in full symbolic form are obtained. Because the expressions of these admittances are too large, we shall present only \( Y_{11} \) expression:

\[
\begin{align*}
Y_{11} &= (C_1^2 + \frac{1}{4} j^2 M - M^2) e + \frac{1}{j} C_1 L C_1 (\frac{1}{R} + j o)^2 + \frac{1}{j} \left[ -2 L C_1^2 + 2 C_1 M - 2 L C_1 C_1 + \frac{2}{j} C_1 R_1 - \frac{1}{j} C_1 C_1 R_1 + C_1 R_1 \right] \frac{1}{j} (R_1, C_1, C_1 \text{ and } M) \\
&+ \frac{1}{j} \left[ -2 L C_1^2 + 2 C_1 M - 2 L C_1 C_1 + \frac{2}{j} C_1 R_1 - \frac{1}{j} C_1 C_1 R_1 + C_1 R_1 \right] \frac{1}{j} \frac{1}{R_1} (R_1, C_1, C_1 \text{ and } M) \frac{1}{j} \frac{1}{M} (M, M_1, M_2, M_3, M_4, M_5) \\
&+ \frac{1}{j} \frac{1}{M} (M, M_1, M_2, M_3, M_4, M_5)
\end{align*}
\]  
\hspace{1cm} (15)

If we denote \( X = \frac{V_1}{V_2} = \frac{A_1}{A_2} e^{j \varphi} \) and take the values of the circuit parameters from Fig. 1, with \( a = 0.0085, b = 0.00071, A_1 = 1.6032 V, \) and \( A_2 = 2.4431 V, \) then (13) becomes a system of two equations with two unknowns, \( X \) and \( \varphi \), so that:

\[
\begin{align*}
X_{\text{analytic}} &= 0.51312 + 0.38539 j \\
\omega_{\text{analytic}} &= 0.6563 \quad \text{rad/s} \\
\Delta \varphi_{21\text{ analytic}} &= \arg \left( X_{\text{analytic}} \right) = 35.97^\circ.
\end{align*}
\]  
\hspace{1cm} (16)

Performing a Spice simulation we get:

\[
\begin{align*}
X_{\text{Spice}} &= 0.6566 \quad \omega_{\text{Spice}} = 36.0^\circ \\
\Delta \varphi_{21\text{ Spice}} &= 35.97^\circ \quad \text{rad/s}.
\end{align*}
\]  
\hspace{1cm} (17)

We can remark the good agreement between these results.

In order to determine the transfer impedances \( Z \) parameters for a two-port passive linear circuit, we connected to the input port (0 - 1) and to the output port (0 - 4) the ideal independent current source \( J_1 (J_1(s)) \) and \( J_2 (J_2(s)) \), respectively, with arbitrary current values, as shown in Fig. 3.
According to the Fig. 2, the transfer impedances are computed with the following relations:

\[
Z_{11}(s) = \frac{V_1(s)}{J_1(s)} \quad ; \quad Z_{12}(s) = \frac{V_1(s)}{J_2(s)} \quad J_{12} = 0
\]

\[
Z_{21}(s) = \frac{V_2(s)}{J_1(s)} \quad ; \quad Z_{22}(s) = \frac{V_2(s)}{J_2(s)} \quad J_{12} = 0
\]

(18)

Running ANCSYANP all transfer impedances in full symbolic form are obtained. Because the expressions of these impedances are too large, we shall present only \(Z_{12}\) expression:

\[
Z_{12} = \left( \frac{C^2 \cdot B_j \left( \frac{m^2 - L^2}{b^2} \right)}{J \cdot C^2 \cdot B_j \left( \frac{m^2 - L^2}{b^2} \right) + 2 \cdot C \cdot R_j} \right) \left( \frac{C \cdot R_j + j \cdot C \cdot B_j \left( \frac{m^2 - L^2}{b^2} \right)}{J \cdot C \cdot R_j + j \cdot C \cdot B_j \left( \frac{m^2 - L^2}{b^2} \right) + 2 \cdot C \cdot R_j} \right)
\]

(19)

The resonant frequencies corresponding to the four transfer impedances have the following expression:

\[
f_{1,2} = \frac{1}{2\pi} \sqrt{\frac{1}{L_1} + \frac{1}{L_2}} = 0.9207 \text{ GHz}; \quad f_{1,2} = 1.019 \text{ GHz}
\]

\[
f_{3,4} = \frac{1}{2\pi} \sqrt{\frac{1}{L_1} + \frac{1}{L_2}} = 0.92085 \text{ GHz}; \quad f_{3,4} = 1.019 \text{ GHz}
\]

(20)

We denote that the resonant frequencies for \(Z_{11}\) and \(Z_{22}\) (\(Z_{12}\) and \(Z_{21}\)) transfer impedances are very closed.

The resonant frequencies for the four transfer admittances have the following expression:

\[
f_{1,2} = 0.859 \text{ GHz}; \quad f_{3,4} = 0.92 \text{ GHz}; \quad f_{1,2} = 0.947 \text{ GHz}; \quad f_{3,4} = 0.947 \text{ GHz}
\]

\[
f_{1,2} = 0.856 \text{ GHz}; \quad f_{3,4} = 0.95 \text{ GHz}; \quad f_{1,2} = 1.025 \text{ GHz}; \quad f_{3,4} = 1.025 \text{ GHz}
\]

(21)

IV. CONCLUSIONS

Using a suitable software we performed the symbolic analysis of an antenna array in order to compute in symbolic form the coupling circuit parameters.

The existing software was enhanced with dedicated routines for generating \(Y\) (admittance), \(Z\) (impedance), \(H\) and fundamental parameters of the coupling network, which is modeled by a passive linear two-port circuit.

The procedure developed can determine all the resonant frequencies of any two-port configuration as functions of the two-port circuit parameters.

The symbolic expressions of the coupling circuit parameters are useful in writing the equations in the sinusoidal behavior (when the nonlinear resistors can be substituted by the linear resistors) of an array of coupled oscillators for the automatic design of these devices.

The program can be used also for a complete analysis in order to compute the phase shift and the synchronous frequency of the antenna array.

The values of the phase shift and the synchronous frequency (analytic expressions) obtained by this approach are in a good agreement with the values obtained by Spice simulation.

REFERENCES


