



# Sensorless Control of a Stepper Motor Based on Higher Order Sliding Modes

Christophe Fiter, Thierry Floquet, Joachim Rudolph

## ► To cite this version:

Christophe Fiter, Thierry Floquet, Joachim Rudolph. Sensorless Control of a Stepper Motor Based on Higher Order Sliding Modes. 8th IFAC Symposium on Nonlinear Control Systems (NOLCOS 2010), Sep 2010, Bologna, Italy. hal-00517870

**HAL Id: hal-00517870**

**<https://hal.science/hal-00517870v1>**

Submitted on 15 Sep 2010

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Sensorless Control of a Stepper Motor Based on Higher Order Sliding Modes

C. Fiter<sup>\*,\*\*</sup> T. Floquet<sup>\*,\*\*</sup> J. Rudolph<sup>\*\*\*</sup>

<sup>\*</sup> *Laboratoire d'Automatique, Génie Informatique et Signal  
(CNRS FRE 3303), Ecole Centrale de Lille,  
59651 Villeneuve d'Ascq, France  
(e-mail: christophe.fiter@centraiens-lille.org,  
thierry.floquet@ec-lille.fr)*

<sup>\*\*</sup> *ALIEN Project team, INRIA, France*

<sup>\*\*\*</sup> *Lehrstuhl für Systemtheorie und Regelungstechnik,  
Universität des Saarlandes, 66123 Saarbrücken, Germany  
(e-mail: j.rudolph@lsr.uni-saarland.de)*

**Abstract:** A robust control for a stepper motor with no position nor velocity sensors and only needing current and voltage measurements is designed. Second order sliding mode based observers are realized to estimate both rotor angular position and velocity. Moreover, a robust control law, which is also based on second order sliding modes and which uses the estimates of the observer, is designed. The stability of the observer based control loop is discussed. The results obtained in simulations indicate the usefulness and the robustness of the method.

**Keywords:** stepping motor, observer, control, stability, robustness, sliding mode.

## 1. INTRODUCTION

Stepper motors are widely used in industry and in consumer goods. In order to design a robust control law for such a motor, the knowledge of the angular position and velocity of the motor is required. However, in many applications, the implementation of mechanical sensors measuring these signals is not desirable. This may be due to the space they require, the wires, their fragility, and also because of non negligible additional cost. For all these reasons, during the last years there has been an increasing interest in designing observers and control laws for the stepper motor which rely on measurements of electrical variables only.

One can find work in the literature which deals with the design of a control supposing that the rotor position is known and using a speed observer (Chiasson et al. [1993], Defoort et al. [2009], Nollet et al. [2008]). Recently, some other work deals with "sensorless" control based on different velocity and position estimation methods, as in Chen et al. [2000], Poulain et al. [2008], and Xu et al. [2007].

The subject of the present work is resumed in Fig. 1. An observer based on second order sliding modes is presented, which provides accurate estimates of the velocity and the position of the stepper motor and uses electric variables measurements only. Then, a robust control law using the estimated variables, based on second order sliding modes and on the notion of flatness (see Fliess et al. [1992]) is designed in order to achieve trajectory tracking. The performance of this control scheme is illustrated by simulation results showing the robustness of the method, even in the presence of perturbation and noise.

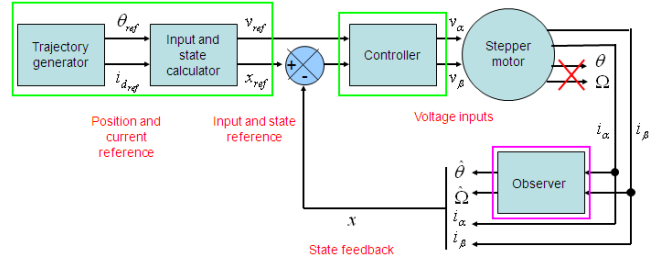


Fig. 1. Sensorless control scheme for the stepper motor

The paper is organized as follows. In Section 2, the different models of the stepper motor that will be used for the observation and the control are displayed. The notion of higher order sliding modes is recalled in Section 3. In Section 4, observers are presented, and in Section 5, a control law based on those observers is analyzed. The results obtained are illustrated by simulations.

## 2. STEPPER MOTOR MODELS

### 2.1 Model in the fixed frame ( $\alpha - \beta$ )

The following model will be used to design the observer:

$$\begin{aligned} \frac{di_\alpha}{dt} &= \frac{1}{L}(v_\alpha - Ri_\alpha + K\Omega \sin N\theta) \\ \frac{di_\beta}{dt} &= \frac{1}{L}(v_\beta - Ri_\beta - K\Omega \cos N\theta) \\ \frac{d\Omega}{dt} &= \frac{1}{J}(K(i_\beta \cos N\theta - i_\alpha \sin N\theta) - f_v\Omega - \tau_L) \\ \frac{d\theta}{dt} &= \Omega \end{aligned} \tag{1}$$

in which  $i_\alpha$  and  $i_\beta$  are the coil currents,  $v_\alpha$  and  $v_\beta$  are the voltage inputs,  $\theta$  is the rotor position, and  $\Omega$  its velocity. The parameter  $R$  represents the resistance,  $L$  the inductance,  $N$  the rotor number of teeth,  $K$  the motor torque constant,  $J$  the rotor moment of inertia,  $f_v$  the coefficient of friction, and  $\tau_L$  the torque load, which will be assumed to be an unknown perturbation.

Only the voltage inputs and the stator currents will be assumed to be measured. Note that the stepper motor without mechanical sensors is observable except for zero velocities.

## 2.2 Model in the moving frame ( $d - q$ )

From the model presented above, one can derive a model in a moving frame, linked to the rotor, thanks to a simple rotation:

$$\begin{bmatrix} i_d \\ i_q \end{bmatrix}^T = M_p \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix}^T \quad \text{with} \quad M_p = \begin{pmatrix} \cos N\theta & \sin N\theta \\ -\sin N\theta & \cos N\theta \end{pmatrix}.$$

Using this change of coordinates, the system (1) becomes

$$\begin{aligned} \frac{di_d}{dt} &= \frac{1}{L}(v_d - Ri_d + NL\Omega i_q) \\ \frac{di_q}{dt} &= \frac{1}{L}(v_q - Ri_q - NL\Omega i_d - K\Omega) \\ \frac{d\Omega}{dt} &= \frac{1}{J}(Ki_q - f_v\Omega - \tau_L) \\ \frac{d\theta}{dt} &= \Omega. \end{aligned} \quad (2)$$

This model will be used to design the control law in Section 5.

## 3. HIGHER ORDER SLIDING MODE

Sliding modes (see Perruquetti et al. [2002]) are often used to design robust nonlinear observers or control laws. The aim of sliding mode control is, by means of a discontinuous control, to robustly constrain the system to reach and stay, after a finite time, on a sliding surface where the resulting behavior has some prescribed dynamics. The sliding surface is defined by the vanishing of a corresponding sliding variable  $s$  and its successive time derivatives up to a certain order, i.e. the  $r^{th}$  order sliding set

$$S_r = \left\{ x \in \mathbb{R}^n : s(x) = \dot{s}(x) = \dots = s^{(r-1)}(x) = 0 \right\}.$$

A control law leading to such a behavior is called a  $r^{th}$  order ideal sliding mode algorithm with respect to  $s$ . Higher order sliding modes, that are characterized by a discontinuous control acting on the  $r^{th}$ ,  $r > 1$ , time derivatives of the sliding vector (instead of the first time derivative in classical sliding mode,  $r = 1$ ), can reduce the chattering phenomenon while preserving the robustness properties.

A wide range of applications using sliding modes for either observation or control, as in mechanics, robotics, or electric machines, can be found in the literature (see for example Bartolini et al. [2003], Butt et al. [2008], Canale et al. [2008], Defoort et al. [2008], Drakunov et al. [2005], Floquet et al. [2003], Martinez et al. [2008], Pisano

et al. [2008], Riachy et al. [2008]). In what follows, higher order sliding mode observers will be designed because they allow one to obtain estimates in finite time without introducing a low-pass filter (which is not the case with a first order sliding mode, for which it is necessary to filter the high frequency oscillations). Especially, the super twisting algorithm (see Levant [2001]) will be used, which is a second order sliding mode defined by

$$u_{st}(s) = u_1(s) + u_2(s) \quad (3)$$

with

$$\begin{cases} \dot{u}_1(s) = -\alpha \text{sign}(s) \\ u_2(s) = -\lambda |s|^{\frac{1}{2}} \text{sign}(s) \end{cases} \quad (4)$$

where  $s$  is the sliding variable, and where  $\alpha$  and  $\lambda$  are two parameters which have to satisfy sufficient conditions (see Levant [2001]) depending on the system and the chosen sliding variable in order to ensure the finite time convergence towards the second order sliding set

$$S_2 = \{x \in \mathbb{R}^n : s(x) = \dot{s}(x) = 0\}. \quad (5)$$

## 4. POSITION AND VELOCITY OBSERVER BASED ON SECOND ORDER SLIDING MODES

In order to design the observer, consider the two electric equations of model (1), and replace the unknown terms by an output injection based on the second order sliding mode algorithm described above. The observer dynamics is described by

$$\begin{aligned} \frac{d\hat{i}_\alpha}{dt} &= \frac{1}{L}[v_\alpha - Ri_\alpha] - \chi_1 \\ \frac{d\hat{i}_\beta}{dt} &= \frac{1}{L}[v_\beta - Ri_\beta] - \chi_2 \end{aligned} \quad (6)$$

The sliding variables are chosen as the current estimation errors:

$$\begin{aligned} \varepsilon_1 &= i_\alpha - \hat{i}_\alpha \\ \varepsilon_2 &= i_\beta - \hat{i}_\beta. \end{aligned}$$

and the output injections  $\chi_1$  and  $\chi_2$  are functions of  $\varepsilon_1$  and  $\varepsilon_2$  given by the super twisting algorithm (3):

$$\begin{aligned} \chi_1 &= u_{st}(\varepsilon_1) \\ \chi_2 &= u_{st}(\varepsilon_2). \end{aligned} \quad (7)$$

The observation error dynamics is obtained using (1) and (6):

$$\begin{aligned} \dot{\varepsilon}_1 &= \frac{K}{L}\Omega \sin N\theta + \chi_1 \\ \dot{\varepsilon}_2 &= -\frac{K}{L}\Omega \cos N\theta + \chi_2. \end{aligned} \quad (8)$$

In order to satisfy the conditions given in Levant [2001] that insure the finite time convergence of the estimation errors onto the sliding set defined in (5), one needs to set the parameters  $\alpha$  and  $\lambda$  from both super twisting algorithms described in (3) and (4):

$$\alpha > C_0 \quad \text{and} \quad \lambda^2 > 4C_0 \frac{\alpha + C_0}{\alpha - C_0},$$

with

$$C_0 = \frac{K}{L}(|\dot{\Omega}|_{max} + N|\Omega^2|_{max}).$$

With these conditions, one has  $\varepsilon_1 = \dot{\varepsilon}_1 = 0$  and  $\varepsilon_2 = \dot{\varepsilon}_2 = 0$  after a finite time. This yields

$$\begin{aligned}\chi_1 &= -\frac{K}{L}\Omega \sin N\theta \\ \chi_2 &= \frac{K}{L}\Omega \cos N\theta.\end{aligned}\quad (9)$$

From the two latter equations, estimates of the rotor position  $\theta$  and velocity  $\Omega$  are given by

$$\begin{aligned}\hat{\Omega} &= \pm \frac{L}{K} \sqrt{\chi_1^2 + \chi_2^2} \\ \hat{\theta} &= \begin{cases} \hat{\theta}_0 & \text{if } \hat{\Omega} = 0, \\ -\frac{1}{N} \arctan\left(\frac{\chi_1}{\chi_2}\right) & \text{if } \hat{\Omega}\chi_2 > 0, \\ \frac{1}{N} \arctan\left(\frac{\chi_1}{\chi_2}\right) & \text{otherwise} \end{cases}\end{aligned}\quad (10)$$

where  $\hat{\theta}_0$  is the latest calculated value of  $\hat{\theta}$  before  $\hat{\Omega} = 0$ .

Two problems caused by the non-uniqueness of the solutions of the electrical system of (1) can be pointed out. The first is that the sign of the velocity is undetermined, since if  $(\theta, \Omega)$  is a solution of (9),  $(\theta \pm \frac{\pi}{N}, -\Omega)$  is a solution too. This also means that if the estimate of the sign of the velocity is not correct, the position estimate will be shifted by  $\pm \frac{\pi}{N}$ . The second problem is that the value of the position estimate is only obtained modulo  $\frac{2\pi}{N}$  because  $\hat{\theta}$  is calculated according to (10) within the interval  $]-\frac{\pi}{N}, \frac{\pi}{N}[$ .

Assuming the sign of the velocity is known, the observer can achieve good performance even in presence of perturbations. However, the two problems mentioned above are still to be solved.

The idea for solving the sign issue is to study the evolution of the position. To this end, the estimate  $\hat{\theta}$  of the position is used and another super twisting algorithm based observer is introduced:

$$\frac{d\hat{\theta}}{dt} = \chi_3(\hat{\theta} - \hat{\theta}) \quad (11)$$

where  $\chi_3$  is given by the super twisting algorithm.

Writing  $\varepsilon_3 = \hat{\theta} - \hat{\theta}$  as the observation error, the error dynamics is:

$$\dot{\varepsilon}_3 = \frac{d\hat{\theta}}{dt} - \chi_3(\varepsilon_3).$$

Setting the parameters for this other super twisting algorithm

$$\alpha > C'_0 \text{ and } \lambda^2 > 4C'_0 \frac{\alpha + C'_0}{\alpha - C'_0},$$

with

$$C'_0 = \left| \frac{d\hat{\Omega}}{dt} \right|_{max}$$

implies the finite time convergence of the observer towards the second order sliding set  $\{\varepsilon_3 = \dot{\varepsilon}_3 = 0\}$  and thus:  $\hat{\theta} = \hat{\theta}$  and  $\hat{\Omega} = \chi_3(\varepsilon_3) = \frac{d\hat{\theta}}{dt}$ .

It has been seen that one has  $\hat{\theta} = \theta \pm \frac{\pi}{N}$  if the estimate of the velocity sign is wrong. In this case, the time derivative of  $\hat{\theta}$  is equal to  $\Omega$ . Then, the output of the third observer  $\hat{\hat{\Omega}}$  converges towards the actual value of the velocity  $\Omega$ . As a consequence, the sign of the velocity estimate  $\hat{\hat{\Omega}}$  is fully

determined, except locally around possible discontinuity points of  $\hat{\theta}$ , for which the sliding mode could be destroyed. Those discontinuity points can create estimation errors for this observer. In order to solve this problem, one can use an algorithm to detect these discontinuity points on the sliding variable  $\varepsilon_3 = \hat{\theta} - \hat{\hat{\theta}}$  (see Fig. 2).

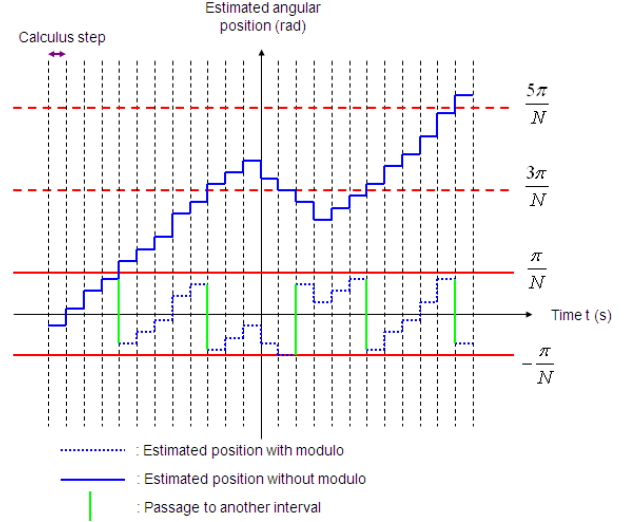


Fig. 2. Discontinuity point detection and position reconstruction algorithm

Such an algorithm to detect the discontinuity points is by construction sensitive to measurement noise: a "jump" caused by the noise could be interpreted as a passage to another interval by the algorithm. However, the super twisting algorithm used for the position observation in the observer (11) plays the role of a filter and provides a continuous position estimate  $\hat{\theta}$ . The results obtained in simulation for a system with large perturbations ( $\tau_L = \frac{\tau_h}{2} \sin(6\pi t)$ , with  $\tau_h$  the motor holding torque) and measurement noise (white noise - up to 5% of the nominal current) highlight the interest of the method (see Fig. 3).

## 5. SENSORLESS CONTROL

In this section, a robust control law without any mechanical sensor is designed using the estimate obtained with the observers defined above.

The control objective is to accurately track a given trajectory. First, note that the system (2) is flat (see Fliess et al. [1992]) with  $z = [\theta, i_d]^T$  as a flat output. This implies that all the system states and inputs can be expressed as functions of a flat output  $z$  and a finite number of its time derivatives. Thanks to this property, one can easily define a reference trajectory (denoted  $\Gamma_r$ ), satisfying the nominal system dynamics without load torque. The dynamics of the tracking error

$$e = [i_d - i_{d_r}, i_q - i_{q_r}, \Omega - \Omega_r, \theta - \theta_r]^T = [e_1, e_2, e_3, e_4]^T$$

is then given by

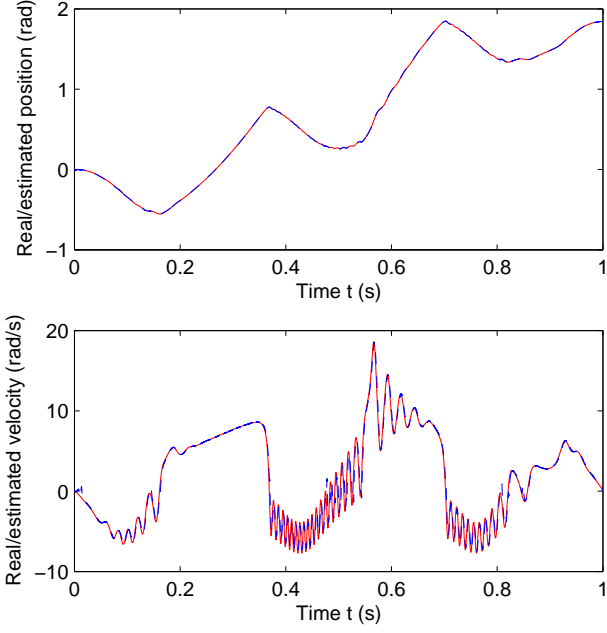


Fig. 3. Real (in red) and estimated (in blue) values of  $\theta$  and  $\Omega$

$$\begin{aligned}
 \dot{e}_1 &= \frac{1}{L}(\bar{v}_d - Re_1 + NL(e_3e_2 + e_3i_{qr} + e_2\Omega_r)) \\
 \dot{e}_2 &= \frac{1}{L}(\bar{v}_q - Re_2 - NL(e_3e_1 + e_3i_{dr} + e_1\Omega_r) - Ke_3) \\
 \dot{e}_3 &= \frac{1}{J}(Ke_2 - f_v e_3 - \tau_L) \\
 \dot{e}_4 &= e_3
 \end{aligned} \tag{12}$$

with  $\bar{v}_d = v_d - v_{dr}$  and  $\bar{v}_q = v_q - v_{qr}$  the new system inputs.

### 5.1 State feedback control

Before designing the sensorless control law, let us recall a second order sliding mode control described in Nollet et al. [2008] that requires the knowledge of the full state.

Two sliding variables are defined as

$$S_\theta = ke_4 + \dot{e}_4 \text{ and } S_{i_d} = e_1, \text{ for } k > 0.$$

It can be seen that the sliding motion  $S_\theta = S_{i_d} = 0$  implies the vanishing of the position tracking error  $e_4$  and the direct current tracking error  $e_1$ .

Because the system has relative degree 2 with respect to  $S_\theta$ , one has to compute the second time derivative of  $S_\theta$  in order to derive the control law used for the tracking of the position:

$$\ddot{S}_\theta = \frac{K}{JL}\bar{v}_q + \mu_\theta(e, \Gamma_r) - \left(\frac{k}{J} - \frac{f_v}{J^2}\right)\tau_L - \frac{1}{J}\frac{d\tau_L}{dt}$$

where

$$\begin{aligned}
 \mu_\theta(e, \Gamma_r) &= \frac{k}{J}(Ke_2 - f_v e_3) - \frac{f_v}{J^2}(Ke_2 - f_v e_3) \\
 &\quad - \frac{K}{JL}(Re_2 + NL(e_3e_1 + e_3i_{dr} + e_1\Omega_r) + Ke_3).
 \end{aligned}$$

Then, a control law based on the sampled twisting algorithm (see Levant [1993]) is considered (it is well suited to systems with relative degree 2):

$$\frac{K}{JL}\bar{v}_q = -\mu_\theta(e, \Gamma_r) + u_{te}(S_\theta, \Delta S_\theta) \tag{13}$$

$$u_{te}(S, \Delta S) = \begin{cases} -\lambda_m \text{sign}(S) & \text{if } S\Delta S \leq 0 \\ -\lambda_M \text{sign}(S) & \text{if } S\Delta S > 0 \end{cases} \tag{14}$$

where  $\Delta S$  is the variation of  $S$  during a sampling period.

To guarantee the finite time convergence in finite time on the surface  $S_\theta = 0$ ,  $\lambda_m$  and  $\lambda_M$  have to verify

$$\begin{aligned}
 \lambda_m &> \left| \left( \frac{k}{J} - \frac{f_v}{J^2} \right) \tau_L - \frac{1}{J} \frac{d\tau_L}{dt} \right|_{max} \\
 \lambda_M &> \lambda_m + 2 \left| \left( \frac{k}{J} - \frac{f_v}{J^2} \right) \tau_L - \frac{1}{J} \frac{d\tau_L}{dt} \right|_{max}
 \end{aligned} \tag{15}$$

Because the system has relative degree 1 with respect to  $S_{i_d}$ , the first time derivative of  $S_{i_d}$  is calculated in order to derive the control law used for the tracking of the direct current:

$$\dot{S}_{i_d} = \dot{e}_1 = \frac{1}{L}\bar{v}_d + \mu_{i_d}(e, \Gamma_r) \tag{16}$$

where

$$\mu_{i_d}(e, \Gamma_r) = \frac{1}{L}(-Re_1 + NL(e_3e_2 + e_3i_{qr} + e_2\Omega_r)).$$

The super twisting algorithm (well suited to systems with relative degree 1) is then used to design the control law

$$\frac{1}{L}\bar{v}_d = -\mu_{i_d}(e, \Gamma_r) + u_{st}(S_{i_d}). \tag{17}$$

Setting the parameters  $\alpha > 0$  and  $\lambda > 0$  of this super twisting algorithm ensures the finite time convergence towards the surface  $S_{i_d} = 0$ .

Therefore, the designed control law guarantees the exponential convergence of the tracking error for a system in which the whole state is measured. It is also shown (see Nollet et al. [2008]) that the system is robust with respect to perturbations and parametric uncertainties.

### 5.2 Observer based control

Suppose now that only the electrical variables are measured and derive a sensorless control law based on the state feedback one described in Section 5.1. Denote the observation error vector as

$$\varepsilon = [i_\alpha - \hat{i}_\alpha, i_\beta - \hat{i}_\beta, \hat{\theta} - \hat{\theta}]^T = [\varepsilon_1, \varepsilon_2, \varepsilon_3]^T.$$

Then, the dynamics of the whole system state, composed of the tracking error and the observation errors

$$\Xi = [e_1, e_2, e_3, e_4, \varepsilon_1, \varepsilon_2, \varepsilon_3]^T$$

is given by

$$\dot{\Xi} = \begin{pmatrix} -\frac{R}{L}e_1 + N(e_3e_2 + e_3i_{qr} + e_2\Omega_r) \\ -\frac{Re_2 + Ke_3}{L} - N(e_3e_1 + e_3i_{dr} + e_1\Omega_r) \\ \frac{1}{J}(Ke_2 - f_v e_3 - \tau_L) \\ e_3 \\ \frac{K}{L}(e_3 + \Omega_r) \sin(N(e_4 + \theta_r)) \\ -\frac{K}{L}(e_3 + \Omega_r) \cos(N(e_4 + \theta_r)) \\ e_3 + \Omega_r \end{pmatrix} + \begin{pmatrix} \frac{1}{L}\bar{v}_d \\ \frac{1}{L}\bar{v}_q \\ 0 \\ 0 \\ \chi_1(\varepsilon_1) \\ \chi_2(\varepsilon_2) \\ \chi_3(\varepsilon_3) \end{pmatrix}. \tag{18}$$

Consider the control given in (13) and (17). The values of the control inputs  $\bar{v}_q$  and  $\bar{v}_d$  have to be adapted to the sensorless case. For this, the following error between the estimated values of the state and the reference values is introduced:

$$\xi = [\hat{i}_d - i_{d_r}, \hat{i}_q - i_{q_r}, \hat{\Omega} - \Omega_r, \hat{\theta} - \theta_r]^T = [\xi_1, \xi_2, \xi_3, \xi_4]^T.$$

Then, one has

$$\begin{aligned} \frac{K}{JL} \bar{v}_q &= -\hat{\mu}_\theta(\xi, \Gamma_r) + u_{te}(\hat{S}_\theta, \Delta \hat{S}_\theta) \\ \frac{1}{L} \bar{v}_d &= -\hat{\mu}_{i_d}(\xi, \Gamma_r) + u_{st}(\hat{S}_{i_d}) \end{aligned} \quad (19)$$

where  $\hat{S}_\theta$  and  $\hat{S}_{i_d}$  are the new sliding variables (similar to  $S_\theta$  and  $S_{i_d}$ )

$$\begin{cases} \dot{\hat{S}}_\theta = k\xi_4 + \dot{\xi}_4 \\ \dot{\hat{S}}_{i_d} = \xi_1 \end{cases}, k > 0$$

and where  $\hat{\mu}_\theta(\xi, \Gamma_r)$  and  $\hat{\mu}_{i_d}(\xi, \Gamma_r)$  are given by

$$\begin{aligned} \hat{\mu}_\theta(\xi, \Gamma_r) &= \frac{k}{J}(K\xi_2 - f_v\xi_3) - \frac{f_v}{J^2}(K\xi_2 - f_v\xi_3) \\ &\quad - \frac{K}{JL}(R\xi_2 + NL(\xi_3\xi_1 + \xi_3i_{d_r} + \xi_1\Omega_r) + K\xi_3) \\ \hat{\mu}_{i_d}(\xi, \Gamma_r) &= \frac{1}{L}(-R\xi_1 + NL(\xi_3\xi_2 + \xi_3i_{q_r} + \xi_2\Omega_r)). \end{aligned}$$

The superposition principle cannot be used to prove the closed-loop system stability because the system is non-linear. Therefore, the convergence of both systems (observer/control), considered separately, does not imply the convergence of the whole system.

In order to prove the exponential stability of the closed-loop system, it will first be shown that the state is bounded in finite time. Then, since the designed observers converge in finite time, after a finite time the system will behave exactly like the system controlled with the state feedback described in the subsection 5.1 and will hence be exponentially stable.

Let us prove that the system (18) is bounded in finite time. To do this, write  $\xi$  as a function of the system state  $\Xi$ , the reference trajectory  $\Gamma_r$ , and the control inputs  $\chi_1(\varepsilon_1)$ ,  $\chi_2(\varepsilon_2)$ ,  $\chi_3(\varepsilon_3)$ ,  $u_{te}(\hat{S}_\theta, \Delta \hat{S}_\theta)$  and  $u_{st}(\hat{S}_{i_d})$ . One has

$$\begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix} = \begin{pmatrix} \hat{i}_d \\ \hat{i}_q \end{pmatrix} - \begin{pmatrix} i_{d_r} \\ i_{q_r} \end{pmatrix}.$$

Since one also has

$$\begin{pmatrix} \hat{i}_d \\ \hat{i}_q \end{pmatrix} = \begin{pmatrix} \cos(N\hat{\theta}) & \sin(N\hat{\theta}) \\ -\sin(N\hat{\theta}) & \cos(N\hat{\theta}) \end{pmatrix} \begin{pmatrix} \hat{i}_\alpha \\ \hat{i}_\beta \end{pmatrix}$$

with

$$\hat{\theta} = -\frac{1}{N} \arctan\left(\frac{\chi_1}{\chi_2}\right), \quad \begin{cases} \hat{i}_\alpha = i_\alpha - \varepsilon_1 \\ \hat{i}_\beta = i_\beta - \varepsilon_2 \end{cases}$$

as well as

$$\begin{pmatrix} i_\alpha \\ i_\beta \end{pmatrix} = \begin{pmatrix} \cos(N(e_4 + \theta_r)) & -\sin(N(e_4 + \theta_r)) \\ \sin(N(e_4 + \theta_r)) & \cos(N(e_4 + \theta_r)) \end{pmatrix} \begin{pmatrix} i_d \\ i_q \end{pmatrix}$$

and

$$\begin{pmatrix} i_d \\ i_q \end{pmatrix} = \begin{pmatrix} e_1 + i_{d_r} \\ e_2 + i_{q_r} \end{pmatrix}$$

then one obtains

$$\begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix} = \Phi(\chi_1, \chi_2) \left( \Psi(e_4 + \theta_r) \begin{pmatrix} e_1 + i_{d_r} \\ e_2 + i_{q_r} \end{pmatrix} - \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \end{pmatrix} \right) - \begin{pmatrix} i_{d_r} \\ i_{q_r} \end{pmatrix} \quad (20)$$

with

$$\Phi(\chi_1, \chi_2) = \begin{pmatrix} \frac{1}{\sqrt{1 + \left(\frac{\chi_1}{\chi_2}\right)^2}} & -\frac{\frac{\chi_1}{\chi_2}}{\sqrt{1 + \left(\frac{\chi_1}{\chi_2}\right)^2}} \\ \frac{\frac{\chi_1}{\chi_2}}{\sqrt{1 + \left(\frac{\chi_1}{\chi_2}\right)^2}} & \frac{1}{\sqrt{1 + \left(\frac{\chi_1}{\chi_2}\right)^2}} \end{pmatrix}$$

and

$$\Psi(e_4 + \theta_r) = \begin{pmatrix} \cos(N(e_4 + \theta_r)) & -\sin(N(e_4 + \theta_r)) \\ \sin(N(e_4 + \theta_r)) & \cos(N(e_4 + \theta_r)) \end{pmatrix}.$$

The variable  $\xi_3$  can also be expressed as

$$\xi_3 = \hat{\Omega} - \Omega_r = \chi_3 - \Omega_r. \quad (21)$$

After substituting in the system (18) the equations of the controls  $\bar{v}_d$  and  $\bar{v}_q$  (19) with the expressions (20) and (21) of the errors between the estimated values and the reference values, one gets a model depending only on the state and the control inputs:

$$\dot{\Xi} = F(\Xi, \Gamma_r, \chi_1, \chi_2, \chi_3, u_{te}(\hat{S}_\theta, \Delta \hat{S}_\theta), u_{st}(\hat{S}_{i_d})).$$

Without loss of generality, it can be assumed that the chosen reference trajectory  $\Gamma_r$  is bounded. Then one has, using (20) and (21)

$$\begin{cases} \xi_1 = O(e_1 + e_2 + \varepsilon_1) \\ \xi_2 = O(e_1 + e_2 + \varepsilon_2) \\ \xi_3 = O(\varepsilon_3). \end{cases} \quad (22)$$

Since the currents  $i_d$  and  $i_q$  are saturated, as well as their estimated values  $\hat{i}_d$  and  $\hat{i}_q$ , the tracking errors of the currents,  $e_1$  and  $e_2$ , as well as the estimation errors  $\varepsilon_1$  and  $\varepsilon_2$  are bounded. Hence,  $\xi_1$  and  $\xi_2$  are also bounded according to (22).

It can be seen that the sampled twisting control input (14) is also bounded, as well as the super twisting algorithms (7), (11) and (17) since they are continuous functions with values in a compact set.

As a consequence, the following inequality can be obtained:

$$\|\dot{\Xi}\| \leq Q\|\Xi\| + \bar{g} \quad (23)$$

where  $Q$  and  $\bar{g}$  are positive constants.

Then, by integrating (23), one gets

$$\|\Xi(t)\| \leq \|\Xi(0)\| + \int_0^t (Q\|\Xi(\tau)\| + \bar{g})d\tau$$

and after applying Grönwall's lemma, one obtains

$$\|\Xi(t)\| \leq \|\Xi(0)\| \exp(Ct) + \frac{\bar{g}}{Q} \exp(Ct - 1)$$

where  $C$  is a positive constant.

Thus, the trajectories of the whole system state  $\Xi$  are bounded in finite time, which concludes the proof of exponential stability of the closed-loop system (18)-(19).

In Fig. 4 are reported simulation results obtained with large perturbations ( $\tau_L = \frac{2}{3}\tau_h \sin(6\pi t)$ , with  $\tau_h$  the motor holding torque), measurement noise (white noise - up to 5% of the nominal current) and parametric errors (5% on parameters  $R$ ,  $L$ ,  $K$ ,  $J$  and  $f_v$ ).



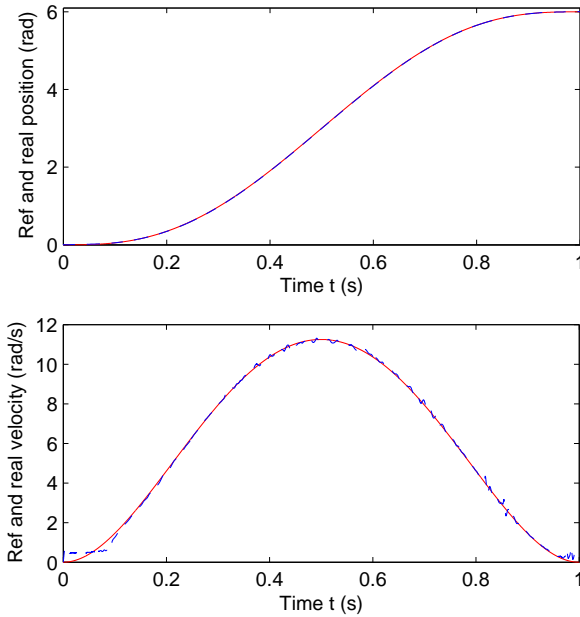


Fig. 4. Reference (in red) and real (in blue) values of  $\theta$  and  $\Omega$  with measurement noise and parametric errors

It can be seen in Fig. 4 that good tracking of both position and velocity is achieved, even in the presence of perturbations, noise and parameter uncertainties.

## 6. CONCLUSION

Higher order sliding mode observers have been designed to estimate the mechanical state of a stepper motor. Then, a robust sensorless control law using those observers has been presented. The results obtained in simulation for the observation scheme as well as the control are convincing and show the potential of the method.

## REFERENCES

- G. Bartolini, A. Pisano, E. Punta, and E. Usai. A survey of applications of second-order sliding mode control to mechanical systems. *International Journal of Control*, volume 76, issue 9–10, pages 875–892, 2003.
- Q.R. Butt, and A.I. Bhatti. Estimation of gasoline-engine parameters using higher order sliding mode. *IEEE Transactions on Industrial Electronics*, volume 55, issue 11, pages 3891–3898, 2008.
- M. Canale, L. Fagiano, A. Ferrara, and C. Vecchio. Vehicle yaw control via second-order sliding-mode technique. *IEEE Transactions on Industrial Electronics*, volume 55, issue 11, pages 3908–3916, 2008.
- Z. Chen, M. Tomita, S. Doki, and S. Okuma. New adaptive sliding observers for position and velocity-sensorless controls of brushless DC motors. *IEEE Transactions on Industrial Electronics*, volume 47, issue 3, pages 582–591, 2000.
- K. Chiasson, and R. Novotnak. Nonlinear speed observer for the PM stepper motor. *IEEE Transactions on Automatic Control*, volume 38, issue 10, pages 1584–1588, 1993.
- M. Defoort, T. Floquet, A. Kökösy, and W. Perruquetti. Sliding mode formation control for cooperative autonomous mobile robots. *IEEE Transactions on Industrial Electronics*, volume 55, issue 11, pages 3944–3953, 2008.
- M. Defoort, F. Nollet, T. Floquet, and W. Perruquetti. A third-order sliding-mode controller for a stepper motor. *IEEE Transactions on Industrial Electronics*, volume 56, issue 9, pages 3337–3346, 2009.
- S. V. Drakunov, T. Floquet, and W. Perruquetti. Stabilization and tracking control for an extended Heisenberg system with a drift, *Systems & Control Letters*, volume 54, pages 435–445, 2005.
- M. Fliess, J. Lévine, P. Martin, and P. Rouchon. Flatness and defect of nonlinear systems: Introductory theory and examples. *International Journal of Control*, volume 61, issue 6, pages 1327–1362, 1995.
- T. Floquet, J.P. Barbot, and W. Perruquetti. Higher order sliding mode stabilization for a class of nonholonomic perturbed systems. *Automatica*, volume 39, issue 6, pages 1077–1083, 2003.
- R. Martinez, J. Alvarez, and Y. Orlov. Hybrid sliding mode-based control of underactuated systems with dry friction *IEEE Transactions on Industrial Electronics*, volume 55, issue 11, pages 3998–4003, 2008.
- A. Levant. Sliding order and sliding accuracy in sliding mode control. *International Journal of Control*, volume 58, issue 6, pages 1247–1263, 1993.
- A. Levant. Universal SISO sliding-mode controllers with finite-time convergence. *IEEE Transactions on Automatic Control*, volume 46, issue 9, pages 1447–1451, 2001.
- F. Nollet, T. Floquet, and W. Perruquetti. Observer-based second order sliding mode control laws for stepper motors. *Control Engineering Practice*, volume 16, issue 4, pages 429–443, 2008.
- W. Perruquetti, and J.P. Barbot. *Sliding mode control in engineering*, Edition Marcel Dekker, New York, 2002.
- A. Pisano, A. Davila, L. Fridman, and E. Usai. Cascade control of PM DC drives via second order sliding mode technique. *IEEE Transactions on Industrial Electronics*, volume 55, issue 11, pages 3846–3854, 2008.
- F. Poulain, L. Praly, and R. Ortega. An observer for permanent magnet synchronous motors with current and voltages as only measurements. *47th IEEE Conference on Decision and Control, CDC2008*, Cancún, Mexico, 2008.
- S. Riachy, Y. Orlov, T. Floquet, R. Santiesteban, and J.P. Richard. Second order sliding mode control of underactuated mechanical systems I: local stabilization with application to an inverted pendulum. *International Journal of Robust Nonlinear Control*, volume 18, issue 4–5, pages 529–543, 2008.
- Z. Xu, and M.F. Rahman. An adaptive sliding stator flux observer for a direct-torque-controlled IPM synchronous motor drive. *IEEE Transactions on Industrial Electronics*, volume 54, issue 5, pages 2398–2406, 2007.