



HAL
open science

SUM RULES AND PION FORM FACTOR IN QCD

V. Nesterenko, A. Radyushkin, A. Efremov

► **To cite this version:**

V. Nesterenko, A. Radyushkin, A. Efremov. SUM RULES AND PION FORM FACTOR IN QCD. Journal de Physique Colloques, 1982, 43 (C3), pp.C3-242-C3-245. 10.1051/jphyscol:1982348 . jpa-00221902

HAL Id: jpa-00221902

<https://hal.science/jpa-00221902>

Submitted on 4 Feb 2008

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

SUM RULES AND PION FORM FACTOR IN QCD

V.A. Nesterenko and A.V. Radyushkin

Presented by A.V. Efremov

Laboratory of Theoretical Physics, JINR, Dubna, USSR

The discovery that perturbative QCD methods can be applied to study the asymptotic behaviour of the pion electromagnetic form factor (see, e.g., refs./1,2/ and the reviews /3/) has been an important step in the development of the theory of hard processes. The pion form factor for all orders and for all logarithms in perturbation theory was shown to have the factorized form:

$$F_{\pi}(Q^2) = \int_0^1 dx \int_0^1 dy \phi^*(y, \mu) E(Q, \mu, x, y, \alpha_s) \phi(x, \mu) + O\left(\frac{1}{Q^4}\right),$$

where $Q^2 = -q^2$, μ is a renormalization parameter, ϕ is the pion wave function and E is the amplitude of the short-distance (SD) subprocess $q\bar{q}^* \rightarrow q'\bar{q}'$ (Fig. 1)

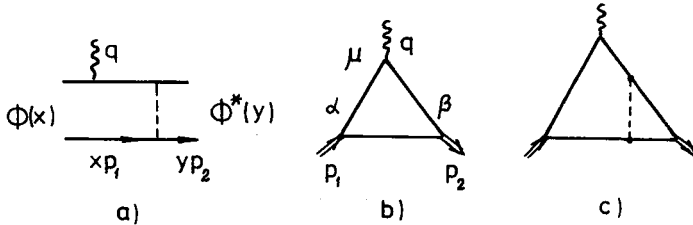


Fig. 1

As it is now generally accepted /4-6/, the best choice for μ is in the region where the higher order PT corrections are small. The 1-loop correction to E have been calculated by Dittes and Radyushkin /7/ ^{*}. Its magnitude depends on the wave function $\phi(x, \mu)$. It was shown /9/, that asymptotically $\phi(x, \mu) \rightarrow 8f_{\pi} x(1-x)$ as $\mu \rightarrow \infty$. However, for $\mu \leq 1$ GeV the wave function should be very broad for nearly-massless strongly interacting quarks. Thus, if one parametrizes $\phi(x)$ simply as $\phi(x) \sim x^r(1-x)^r$, then one should expect that for pion $r \ll 1$ (say, $r \approx 0.1 - 0.3$). As a result, for the naive choice $\mu = Q$ and small r the correction is huge:

$$F_{\pi}(Q) = F_{\pi}^{(0)}(Q) \left\{ 1 + \frac{\alpha_s}{2\pi} \cdot \frac{4}{3} \left[\frac{3}{r^2} - \frac{2}{r} \ln \frac{Q^2}{\mu^2} + O\left(\frac{1}{r}\right) \right] \right\}.$$

^{*}) A similar calculation was performed by Field et al. /8/, but their results for some diagrams are incorrect.

To reduce it, one should take $\mu^2 = \mu_{opt}^2 = Q^2 \exp(-3/2r)$ ($\leq 0.007 Q^2$). This means that for $Q^2 \leq 10 \text{ GeV}^2$ the exchanged gluon (Fig.1a) has virtuality much smaller than 1 GeV^2 . In such a situation it is, of course, misleading to rely on the asymptotic freedom.

First attempt to find the way out was a QCD inspired model /10,4/ with the result that for moderately large Q^2 the pion EM form factor is dominated by the Feynman mechanism /11/. The aim of this report is a brief description of a new approach /12,13,14/ that enables one to analyse the behaviour of $F_\pi(Q^2)$ for moderately large Q^2 .

Technically our analysis is based on the QCD sum rule approach /15/. We consider the three-point amplitude

$$T_{\mu\alpha\beta}(p_1, p_2) = i^2 \int e^{-ip_1x + ip_2y} \langle 0 | T \{ j_\alpha^+(x) J_\mu(0) j_\beta^-(y) \} | 0 \rangle dx dy$$

(see for notation Fig. 1b) where J_μ is the electromagnetic current and $j_\alpha = \bar{d} \gamma_5 \gamma_\alpha u$ is the axial one: $\langle 0 | j_\alpha(0) | P \rangle = i f_\pi P_\alpha$, where $|P\rangle$ is the 1-pion state and $f_\pi \approx 133 \text{ MeV}$ the pion decay constant.

The invariant amplitudes T_i (corresponding to various structures present in $T_{\mu\alpha\beta}$) depend on 3 variables: $p_1^2, p_2^2, q^2 = (p_1 - p_2)^2$. Owing to the asymptotic freedom one may calculate $T_i(p_1^2, p_2^2, q^2)$ in the Euclidean region $p_1^2, p_2^2, q^2 \leq -1 \text{ GeV}^2$. To extract the desired information about the physical states, we use the dispersion relation

$$T(p_1^2, p_2^2, q^2) = \frac{1}{\pi^2} \int_0^\infty \frac{ds_1}{s_1 - p_1^2} \int_0^\infty \frac{ds_2}{s_2 - p_2^2} \rho(s_1, s_2, q^2)$$

and apply

the Borel procedure /15/ in p_1^2 and p_2^2 : $T_i(p_1^2, p_2^2, Q^2) \xrightarrow{\text{B.T.}} \Phi(M_1^2, M_2^2, Q^2)$.

For the pion form factor analysis the most important is the invariant amplitude Φ , related to the structure $P_\alpha P_\beta P_\mu$, where $P = p_1 + p_2$. Neglecting quark masses ($m_{u,d} \leq 10 \text{ MeV}$) we calculated the contribution of the diagram 1b into the ρ density:

$$\rho_0(s_1, s_2, q^2) = \frac{3Q^4}{2} \left\{ \left(\frac{d}{dQ^2} \right)^2 + \frac{Q^2}{3} \left(\frac{d}{dQ^2} \right)^3 \right\} [(s_1 + s_2 + Q^2)^2 - 4s_1 s_2]^{-1/2}. \quad (1)$$

In the real world $\rho(s_1, s_2, q^2)$ differs, of course, from $\rho_0(s_1, s_2, q^2)$ especially for small s_1, s_2 . In particular, it contains the pion term $\rho_\pi(s_1, s_2, q^2) = \pi^2 f_\pi^2 F_\pi(Q^2) \delta(s_1 - m_\pi^2) \delta(s_2 - m_\pi^2)$. The basic idea of the QCD sum rule approach /15/ is that the difference $\Phi - \Phi_0$ is mainly due to power corrections^{x)} $(1/M^2)^N$ generated by nonzero vacuum expectation values of quark and gluon operators. Taking into account the contributions proportional to $\langle G_{\mu\nu}^a G_{\mu\nu}^a \rangle$ and $\alpha_s \langle \bar{q}q \rangle^2$ (cf. ref./15/) we arrive at the following representation for the pion form factor

$$f_\pi^2 F_\pi(Q^2) = \frac{1}{\pi^2} \int_0^{s_0} ds_1 \int_0^{s_0} ds_2 \rho_0(s_1, s_2, q^2) \exp\left(-\frac{s_1 + s_2}{M^2}\right) + \frac{\alpha_s \langle G_{\mu\nu}^a G_{\mu\nu}^a \rangle}{12\pi M^2} + \frac{208\pi\alpha_s \langle \bar{q}q \rangle^2}{81 M^4} \left(1 + \frac{2}{13} \frac{Q^2}{M^2}\right) + O\left(\frac{\alpha_s}{\pi}\right), \quad (2)$$

where $(\alpha_s/\pi) \langle GG \rangle = 0.012 \text{ GeV}^4$, $\alpha_s \langle \bar{q}q \rangle^2 = 1.83 \cdot 10^{-6} \text{ GeV}^2$ /15/ and s_0 is the "effective threshold" characterizing the region where one can substi-

x) To treat initial and final states on equal footing we take henceforth $M_1 = M_2 = M$.

tute $\rho(s_1, s_2, q^2)$ by its free-quark value $\rho_0(s_1, s_2, q^2)$.

It is natural to define that the "true" s_0 is that for which the region of weak sensitivity to variations of unphysical parameter M is the most broad. For $Q^2 = 1-3 \text{ GeV}^2$ this gives $s_0 = 0.6-0.8 \text{ GeV}^2$ respectively (Fig. 2) which is in good agreement with the value $s_0 = 0.7 \text{ GeV}^2$ extracted from the analysis /15/.

To get a reliable result for $\mathcal{F}_\pi(Q^2)$ one should choose M^2 so as to reduce to a minimum both the power corrections (blowing up as $M^2 \rightarrow 0$) and the contribution of states with $s_1, s_2 > s_0$. In our case, however, the situation is more complicated than in /15/; mainly because the free-quark term in eq. (2) behaves like $1/Q^4$ for large Q^2 while the power corrections grow with Q^2 . In particular to reduce the power corrections to 30% for $Q^2 = 2 \text{ GeV}^2$ one should take $M^2 \geq 2 \text{ GeV}^2$. The contribution of higher states (estimated by use of the free-quark density) is then ≈ 100 . However, it is the difference between the true higher state contribution and its free-quark approximation that matters, which may well be even zero for an appropriate

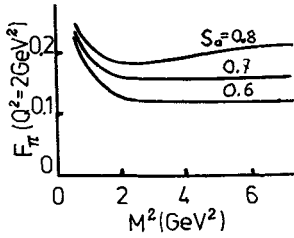


Fig. 2

choice of s_0 . As is seen from Fig.3, in the region $1 < Q^2 < 4 \text{ GeV}^2$ our theoretical curve (broken line) agrees with experimental data /16/ within 10-20% accuracy.

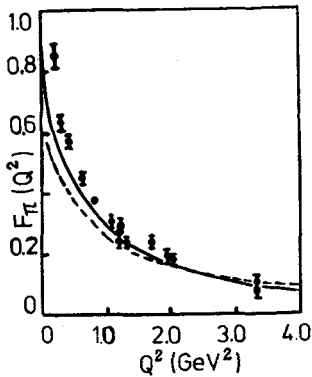


Fig. 3

Using (1) one can reduce eq. (3) to

$$\mathcal{F}_\pi(Q^2) = \frac{s_0}{4\pi^2 f_\pi^2} \left\{ 1 - \frac{1 + 6s_0/Q^2}{(1 + 4s_0/Q^2)^{3/2}} \right\}. \tag{4}$$

For $s_0 = 4\pi^2 f_\pi^2 = 0.7 \text{ GeV}^2$ this formula is in excellent agreement with experimental data (fig. 3, solid line). The agreement indicates that the local duality between the pion and free-quark contributions is indeed a very good approximation.

In principle, apart from power corrections one must take into account also higher perturbative corrections due to diagrams like that shown in Fig. 1c. However, their contribution is damped by the $a_s/\pi \approx 0.1$ factor with respect to that of Fig. 1b. Our estimates show that for $Q^2 < 10 \text{ GeV}^2$ these diagrams produce only 10% (positive) correction to $\mathcal{F}_\pi(Q^2)$. Thus, the celebrated one-gluon-exchange contribution is of little importance for available Q^2 . However, the situation changes drastically in the asymptotic $Q^2 \rightarrow \infty$ region; because Fig. 1c gives in this region $1/Q^2$ -contribution (corresponding to the asymptotic QCD analysis /1-3/) exceeding the $1/Q^4$ -contributions of Fig. 1b for sufficiently large Q^2 .

Taking M^2 sufficiently large one can control the power corrections for arbitrarily high Q^2 values. The most radical way out is to take $M^2 = \infty$. Then the power corrections are absent altogether, and if one assumes that the "effective threshold" s_0 is Q^2 -independent, one obtains the finite energy sum rule

$$\begin{aligned} \frac{1}{\pi^2} \int_0^{s_0} ds_1 \int_0^{s_0} ds_2 \rho_0(s_1, s_2, q^2) &= \\ &= \frac{1}{\pi^2} \int_0^{s_0} ds_1 \int_0^{s_0} ds_2 \rho_\pi(s_1, s_2, q^2) \equiv f_\pi^2 \mathcal{F}_\pi(Q^2). \end{aligned} \tag{3}$$

Eq. (3) is nothing else but a duality relation between the pion and free-quark contributions (cf. /17/ and references therein).

References

1. A.V.Radyushkin, JINR preprint P2-10717 (June, 1977);
A.V.Efremov and A.V.Radyushkin. Phys.Lett. 94B, 2451 (1980)
and references therein.
2. S.J.Brodsky and G.P.Lepage. Phys.Rev., D22, 157 (1980).
3. V.L.Chernyak. Proc. XV LINP Winter School, Leningrad (1980),1,
p. 65;
A.H.Mueller. Phys.Reports, 73, 237 (1981).
4. A.V.Efremov and A.V.Radyushkin, JINR preprint E2-80-521 (1980),
submitted to Madison Conference.
5. P.M.Stevenson. Phys.Rev., D23, 2916 (1981).
6. W.Celmaster and D.Sivers. Phys.Rev. D23, 227 (1981).
7. F.-M.Dittes and A.V.Radyushkin. Yad.Fiz., 34, 529 (1981).
8. R.D.Field et. al. Nucl.Phys., B186, 429 (1981).
9. A.V.Efremov and A.V.Radyushkin, JINR preprint E2-11983,
(October, 1978).
10. A.V.Radyushkin. JINR report P2-80-687 (1980); see also Proc.
of the Symp. on Particle Physics, Visegrad, Hungary (September,
1981), p. 79.
11. R.P.Feynman. Photon-Hadron Interactions. Benjamin (1972).
12. V.A.Nesterenko and A.V.Radyushkin. Pis ma v JETF, 35, 395 (1982).
13. V.A.Nesterenko and A.V.Radyushkin. JINR preprint E2-82-204 (1982),
to be published in Phys.Lett. B.
14. B.L.Ioffe and A.V.Smilga. Preprint ITEP-27 (1982).
15. M.A.Shifman, A.I.Vainshtein and V.I.Zakharov, Nucl.Phys.,
B147, 447 (1979).
16. C.Bebek et al. Phys.Rev. D17, 1693 (1978).
17. N.V.Krasnikov and A.A.Pivovarov. Report NBI-HE-81-38 (Kopenhagen)