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Short Communication

Dynamics of interface depinning in a disordered medium

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Abstract . — The dynamics of a driven interface in a disordered medium close to the depinning threshold is analyzed. By a functional renormalization group scheme exponents characterizing the depinning transition are obtained to the first order in $\epsilon = 4 - D > 0$, where D is the interface dimension. At the transition, the dynamics is superdiffusive with a dynamical exponent $z = 2 - 2\epsilon/9 + O(\epsilon^2)$, and the interface height difference over a distance L grows as L^ζ with $\zeta = \epsilon/3 + O(\epsilon^2)$. The interface velocity in the moving phase vanishes as $(F - F_c)^\theta$ with $\theta = 1 - \epsilon/9 + O(\epsilon^2)$ when the driving force F approaches its threshold value F_c .

The driven viscous motion of an interface in a medium with random pinning forces is one of the paradigms of condensed matter physics. This problem arises, e.g., in the domain-wall motion of a magnetically or structurally ordered system with random-bond or random-field disorder [1], or when an interface between two immiscible fluids is pushed through a porous medium [2]. Closely related problems include impurity pinning in type-II superconductors [3] and in charge-density-wave (CDW) systems [4]. Despite its importance this problem is largely unsolved although a number of attempts have been made in the past (see e.g. [5-10]).

In this paper we focus on a simple realization of the problem, the motion of a D -dimensional interface profile $z(\mathbf{x}, t)$ obeying the following equation [5-8],

$$\lambda \frac{\partial z}{\partial t} = \gamma \nabla^2 z + F + \eta(\mathbf{x}, z). \quad (1)$$

Here λ is the friction (or inverse mobility) coefficient, γ is the stiffness constant, and F is the driving force. The random force $\eta(\mathbf{x}, z)$ is Gaussian distributed with $\langle \eta \rangle = 0$ and

$$\langle \eta(\mathbf{x}_0, z_0) \eta(\mathbf{x}_0 + \mathbf{x}, z_0 + z) \rangle = \delta^D(\mathbf{x}) \Delta(z). \quad (2)$$

We will be mainly concerned with the random-field case where the correlator $\Delta(z) = \Delta(-z)$ is a monotonically decreasing function of z for $z > 0$ and decays rapidly to zero over a finite

distance a . Unless otherwise specified, the width of the correlator (2) along the interface is taken to be much smaller than any other characteristic length of the problem.

As pointed out by Bruinsma and Aeppli [6], an important length of this model is $L_c = [\gamma^2 a^2 / \Delta(0)]^{1/\epsilon}$, where $\epsilon = 4 - D$. For $D < D_c = 4$, the interface is kept smooth (i.e., fluctuations in z is limited to a or smaller) on length scales $L < L_c$, but is able to explore the inhomogeneous force field on larger length scales. It follows that the maximum pinning force on a piece of interface of linear dimension $L > L_c$ is of the order of $(L/L_c)^D [\Delta(0)L_c^D]^{1/2}$, which leads to the estimate $F_c \simeq [\Delta(0)/L_c^D]^{1/2} \sim \Delta(0)^{2/\epsilon}$ for the critical driving force of a depinning transition [5, 6]. For $F > F_c$ we expect a steady-state moving solution to (1) while for $F < F_c$ the interface at long times is pinned at one of the presumably infinitely many locally stable configurations. The nature of the depinning transition at $F = F_c$ has not been studied in detail analytically. [The situation is different for $D > 4$, where pinning is essentially a small length scale phenomenon where mean-field theory is expected to be valid. This conclusion, which can be drawn by assessing the relative importance of the elastic and random force terms in (1), sets the critical dimension at $D_c = 4$ for weak disorder [6].]

Our main purpose here is to develop a renormalization group approach to the critical dynamics near the depinning threshold for $D \leq 4$. A straightforward extension of the perturbation theory of Efetov and Larkin [4] yields a vanishing mobility $\lambda^{-1} = 0$ at L_c , thereby freezes dynamics on larger length scales. However, as we demonstrate below, this difficulty can be overcome by considering the functional renormalization of $\Delta(z)$ which becomes singular at the origin, thus opening the door to a systematic expansion in $D = 4 - \epsilon$ dimensions. We present the first results so obtained for the critical exponents characterizing the depinning transition. Details of our calculation will be presented elsewhere.

The usual perturbation theory consists of expanding $\eta(\mathbf{x}, z)$ at a flat interface (or, for that matter, any other reference configuration), and solving the resulting equation order by order in the strength of the disorder [4, 5, 7]. Such a procedure is justified when deviations from the reference interface position are of order a or smaller. This is indeed the case for a fast moving interface with $D > 2$ which is the starting point of our discussion. (For $D \leq 2$ the related Edwards-Wilkinson equation [11] yields a rough interface. We believe that this is the origin for the break down of perturbation theory as observed by Koplik and Levine [7].)

For a moving interface, we write $z(\mathbf{x}, t) = vt + h(\mathbf{x}, t)$, where v is determined self-consistently from the condition $\langle h(\mathbf{x}, t) \rangle = 0$. Equation (1) now takes the form

$$\lambda \frac{\partial h}{\partial t} = \gamma \nabla^2 h + F - \lambda v + \eta(\mathbf{x}, vt + h(\mathbf{x}, t)). \quad (3)$$

Due to the coupling among different Fourier modes through the random force term, the system's response to a long-wavelength, slow-varying external perturbation is described by a diffusion equation with modified parameters $\lambda_{\text{eff}} = \lambda + \delta\lambda$ and $\gamma_{\text{eff}} = \gamma + \delta\gamma$. At large v the lowest order corrections can be easily found from the perturbation theory,

$$\delta\gamma^{(1)} = \gamma g \frac{1}{D} L_v^\epsilon, \quad \delta\lambda^{(1)} = -\lambda g \frac{1}{\epsilon} L_v^\epsilon, \quad (4)$$

where $L_v = (\gamma a / v \lambda)^{1/2}$ is the diffusion length over a time period a/v and $g = c \Delta''(0) + O(\epsilon)$ is the coupling constant, with $c = 1/(8\pi^2 \gamma^2)$. Here and below $\epsilon > 0$. Result for $\delta\lambda^{(1)}$ in (4) is identical to a previous one by Feigel'man who considered the correction to the velocity $v = F/\lambda$ due to the pinning force [5]. The width of the interface to this order is given by

$$\langle h^2 \rangle = a^2 \frac{4}{(D-2)(4\pi)^{D/2}} \frac{1}{\epsilon} \left(\frac{L_v}{L_c} \right)^\epsilon + O(\epsilon). \quad (5)$$

From equation (4) we see that both corrections are related to the second derivative of $\Delta(z)$ at the origin, and that $\delta\gamma/\gamma$ is by a factor ϵ/D smaller than $\delta\lambda/\lambda$. Extending the above calculation to the next order yields

$$\lambda_{\text{eff}} = \lambda \left[1 + \delta\lambda^{(1)}/\lambda + 2(\delta\lambda^{(1)}/\lambda)^2 + \dots \right], \tag{6}$$

where $\delta\lambda^{(1)}$ is given in (4). Only leading order terms in ϵ in each order are shown in (6). It is apparent that the series obtained from perturbation theory, while valid for large v , cannot be used directly near the pinning threshold, where $v \rightarrow 0$.

The usual ϵ -expansion scheme allows one to sum up the series *via* renormalization group flow equations. Specifically, we consider λ , γ , $F - \lambda v$, and Δ to be renormalizable quantities which depend on the upper cut-off length, $L = L_v$. The flow equations can be immediately read off from (4) and (6),

$$d \ln \gamma / d \ln L = O(\epsilon^2), \tag{7a}$$

$$d \ln \lambda / d \ln L = -g L^\epsilon, \tag{7b}$$

$$dg / d \ln L = -3g^2 L^\epsilon. \tag{7c}$$

The last equation can also be obtained directly from the diagrammatic technique of Larkin and Efetov [4], and is in fact only one of the set of flow equations one of us [12] obtained for the coefficients in a Taylor expansion for $\Delta(z)$. This set of equations can be expressed in the functional form

$$\frac{d \Delta(z)}{d \ln L} = -c L^\epsilon \frac{d^2}{dz^2} \left[\frac{1}{2} \Delta^2(z) - \Delta(z) \Delta(0) \right]. \tag{7d}$$

Interestingly, equation (7d) appears also in the treatment of an equilibrium interface in a random system by Daniel Fisher [13], and in his recent work with Narayan on sliding charge-density-waves in a random medium [14].

Equation (7c) can be integrated to give

$$g(L) = \frac{g_0}{1 + (3/\epsilon)g_0(L^\epsilon - L_0^\epsilon)}, \tag{8}$$

where $g_0 = g(L_0)$. A negative g_0 , which appears to be a natural choice if an analytic $\Delta(z)$ is assumed at L_0 , leads to a diverging $g(L)$ and hence $\Delta''(0)$ at a finite length $L \simeq (\epsilon/3|g_0|)^{1/\epsilon} \simeq L_c$. Inserting (8) into (7b) then yields an infinite λ at $L \simeq L_c$, beyond which no dynamics is possible. On the face of it, this result is clearly unphysical.

Such a diverging behavior has actually been noted earlier in the study of impurity pinning of charge-density-waves by Efetov and Larkin [4], and in other related problems, and its implication remains controversial. One opinion is to dub the pole an artifact of the one-loop approximation bearing no real physical significance. The second and much more interesting proposal is to accept the divergence as a real phenomenon associated with the nonanalytic behavior of $\Delta(z)$ at the origin, and try to continue the renormalization procedure [13]. In the remaining part of the paper we explore consequences of the latter approach and show that it indeed leads to a consistent renormalization scheme and to fruitful results.

The divergence of g corresponds to a singularity of $\Delta(z)$ at the origin. Nevertheless, equation (7d) is still well defined away from $z = 0$ and can thus be followed. To look for a fixed point solution, we make the scaling ansatz

$$\Delta(L, z) = c^{-1} A^{2/3} L^{-\alpha} \Delta_L^*(z A^{-1/3} L^{-\zeta}), \tag{9}$$

where $\lim_{L \rightarrow \infty} \Delta_L^*(y) = \Delta^*(y)$, $a = \epsilon - 2\zeta$, and A is chosen such that $\Delta^*(0) = 1$. As for Δ , Δ^* is an even function of its argument. Inserting (9) into (7d) and taking the limit $L \rightarrow \infty$ yields,

$$(\epsilon - 2\zeta)\Delta^*(y) + \zeta y \Delta^{*\prime}(y) - [\Delta^{*\prime}(y)]^2 - \Delta^{*\prime\prime}(y)[\Delta^*(y) - 1] = 0. \tag{10}$$

Examining the behavior of (10) at small y shows that two types of singular behavior are possible. In the first case we have $\Delta^*(y) = 1 + a_1|y|^{1/2} + O(|y|)$. This form yields a diverging second derivative as $y \rightarrow 0$, thus is not a way out of the difficulty. The second possibility is

$$\Delta^*(y) = 1 + a_1|y| + \frac{1}{2}a_2y^2 + \dots \tag{11}$$

with $a_1^2 = \epsilon - 2\zeta$ and $a_2 = (\epsilon - \zeta)/3$. Here $\Delta^{*\prime\prime}(0+) = a_2$ is finite. Using an expansion of the type (11) for $\Delta(z)$, we find that equations (7a) and (7b) [but not (7c)] remain valid to the first order in ϵ , with the understanding that $g = c\Delta^{*\prime\prime}(0+)$. The singular term $|z|$, however, yields a reduction of the driving force, $F \rightarrow f = F - F_c$. Here $F_c \simeq -(16\pi^2\gamma)^{-1}\Lambda_0^{D-2}\Delta'(L_0, 0+)$ depends on the lower cutoff $\Lambda_0 \simeq \pi/L_0$ of the momentum space integration. Using the scaling form (9) for Δ and identifying L_0 with L_c yields an F_c in agreement with the estimate given in references [5] and [6].

Integrating (7b) using (9) and (11) yields

$$\lambda(L) = \lambda_0(L/L_0)^{-(\epsilon-\zeta)/3}, \tag{12}$$

where $\lambda_0 = \lambda(L_0)$. As before, γ has no scale dependence to the first order in ϵ . Performing the scale transformation $\mathbf{x} \rightarrow b\mathbf{x}$, $t \rightarrow b^z t$, and $h \rightarrow b^\zeta h$, equation (3) can be rewritten as

$$\lambda b^{2-z} \frac{\partial h}{\partial t} = \gamma \nabla^2 h + b^{2-\zeta}(f - \lambda v) + b^{2-\zeta} \eta(b\mathbf{x}, vb^z t + b^\zeta h). \tag{13}$$

Here z is the dynamical exponent to be distinguished from the interface coordinate $z(\mathbf{x}, t)$. It follows from equations (2), (9), and (12) that (13) becomes scale invariant at $v = f = 0$ upon the choice $z = 2 - (\epsilon - \zeta)/3$. A finite v , however, changes the character of the noise correlator above a length scale $L_v \sim v^{-1/(z-\zeta)}$, as can be seen by comparing the two terms in the second argument of η in (13). Physically, L_v serves as the correlation length of the net pinning (or driving) force along the interface. Stop the renormalization at L_v yields $f = \lambda(L_v)v$ which in turn gives

$$v \sim f^\theta, \quad \text{with } \theta = 1 - \frac{1}{3} \frac{\epsilon - \zeta}{2 - \zeta}, \tag{14a}$$

$$L_v \sim f^{-\nu}, \quad \text{with } \nu = 1/(2 - \zeta), \tag{14b}$$

where we have used (12) and the relation between z and ζ . [The condition $L_v \simeq v^{-1/(z-\zeta)}$ alone yields the scaling relation $\theta = \nu(z - \zeta)$ which is satisfied in the present case.]

Our final task is to determine the exponent ζ from (10). For this purpose it is useful to consider the integrals $I_\Delta = \int_{-\infty}^{\infty} \Delta(L, z) dz$, which is an invariant of the flow equation (7d), and $I^* = \int_{-\infty}^{\infty} \Delta^*(y) dy$. For $I_\Delta > 0$, which is true for random-field disorder, we have i) $\zeta < \epsilon/3$, $I^* = \infty$; ii) $\zeta = \epsilon/3$, $I^* = cA^{-1}I_\Delta$; and iii) $\zeta > \epsilon/3$, $I^* = 0$; Case i) is inconsistent with (10) if we demand $\Delta^*(y)$ to be bounded for all y and vanish at infinity. One can also show from the flow equation (7d) that, if $\Delta(L, z)$ is initially positive everywhere and decays to zero

sufficiently fast at large z , the limiting form Δ^* has no negative parts thus excluding iii). In case ii) there is actually a unique solution with exponential tails given implicitly by [13]

$$\Delta^* \exp(-\Delta^*) = \exp\left(-1 - \frac{\epsilon}{6} y^2\right). \quad (15)$$

Inserting $\zeta = \epsilon/3$ in equations (14) and in the expression for z we have the following results for the exponents to the first order in ϵ ,

$$\zeta \simeq \frac{1}{3}\epsilon, \quad z \simeq 2 - \frac{2}{9}\epsilon, \quad \theta \simeq 1 - \frac{1}{9}\epsilon, \quad \nu \simeq \frac{1}{2} + \frac{1}{12}\epsilon. \quad (16)$$

Note that for random-bond disorder, where η can be written as the derivative of a random potential with short-range correlations, $I_\Delta = 0$ and hence the exponents can be quite different from those given in (16).

Let us conclude by recapturing the main steps that led to a successful renormalization scheme for the interface dynamics at and above the depinning transition. We have shown that a simple extension of the perturbation theory carried out to the lowest order runs into difficulty on the length scale L_c where pinning effects become significant. When the renormalization procedure is extended to the whole function $\Delta(z)$ (random force correlator in the moving direction), the divergent behavior of the coupling constant can be attributed to the nonanalyticity of $\Delta(z)$ at the origin. By isolating the singular term the divergence can be formally removed and a consistent renormalization scheme is found at the transition $F = F_c$. The interface roughness exponent $\zeta = \epsilon/3 + O(\epsilon^2)$ so obtained differs, to the first order in ϵ , from the value $\epsilon/2$ from perturbation theory [4, 8], but coincides with that of the equilibrium random-field problem, though we see no *a priori* reason here for an Imry-Ma type argument [1] to exclude higher order corrections. A finite interface velocity $v \sim (F - F_c)^\theta$ interrupts the renormalization process at a length scale $L_v \sim (F - F_c)^{-\nu}$, above which one crosses over to the regime where the random forces act independently on the moving interface as in the Edwards-Wilkinson equation. It is interesting to note that, using our expressions (16) at $D = 1$ yields the temporal roughness exponent $\beta = \zeta/z = 3/4$, in surprisingly good agreement with simulation results of Parisi [8] on a lattice version of (1). We mention here that a self-consistency argument similar to the one given by Harris for equilibrium disordered systems [15] yields an inequality

$$1/\nu \leq (D + \zeta)/2, \quad (17)$$

if a sharp threshold is to be assumed. In our case (17) is fulfilled as an equality to the first order in ϵ .

On the dynamical side, the effect of random forces on the interface motion is qualitatively different below and above L_c . For $L < L_c$ the interface is slowed down but not "pinned", i.e., it responds to an arbitrarily weak driving force with an increased λ . In contrast, the case $L > L_c$ is characterized by a threshold dynamics, i.e., only a sufficiently large driving force $F > F_c$ yields a response. As our calculation shows, F_c is formally related to the amplitude of the singular part of the correlator $\Delta(z)$ at the origin. It is believed that (e.g. Ref. [9]), at $F = F_c$, the system becomes critical in the sense that a small perturbation on a length scale $L > L_c$ may provoke an arbitrarily large response, as in the sandpile model of Bak, Tang and Wiesenfeld [16]. Our finding of a dynamical exponent $z = 2 - 2\epsilon/9 < 2$ suggests that the dynamics at the depinning transition is indeed superdiffusive. It would be interesting to explore the use of functional renormalization group approach to other systems characterized by a threshold dynamics.

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References

- [1] For recent reviews on the closely related equilibrium problem see, e.g., Nattermann T. and Rujan P., *Int. J. Mod. Phys. B* **3** (1989) 1597;
Forgacs G., Lipowsky R. and Nieuwenhuizen Th. M., in Phase transitions and critical phenomena, C. Domb and J. L. Lebowitz Eds. Vol 14 (Academic Press, London, 1991) p.135.
- [2] Rubio M. A., Edwards C. A., Dougherty A. and Gollub J. P., *Phys. Rev. Lett.* **63** (1989) 1685.
- [3] Larkin A. I. and Ovchinnikov Yu. N., *J. Low Temp. Phys.* **34** (1979) 409.
- [4] Efetov K. B. and Larkin A. I., *Sov. Phys. JETP* **45** (1977) 1236.
- [5] Feigel'man M. V., *Sov. Phys. JETP* **58** (1983) 1076.
- [6] Bruinsma R. and Aeppli G., *Phys. Rev. Lett.* **52** (1984) 1547.
- [7] Koplik J. and Levine H., *Phys. Rev. B* **32** (1985) 280;
Kessler D. A., Levine H. and Tu Y., *Phys. Rev. A* **43** (1991) 4551.
- [8] Parisi G., *Europhys. Lett.* **17** (1992) 673.
- [9] Martys N., Cieplak M. and Robbins M. O., *Phys. Rev. Lett.* **66**, (1991) 1058;
Martys N., Robbins M. O. and Cieplak M., *Phys. Rev. B* **44** (1991) 12294.
- [10] Tang L.-H. and Leschhorn H., *Phys. Rev. A* **45** (1992) R8309;
Buldyrev S.V., Barabási A.-L., Caserta F., Havlin S., Stanley H.E. and Vicsek T., *Phys. Rev. A* **45** (1992) R8313.
- [11] Edwards S.F. and Wilkinson D.R., *Proc. R. Soc. London, Ser. A* **381** (1982) 17.
- [12] Stepanow S., submitted to *Ann. Physik*.
- [13] Fisher D.S., *Phys. Rev. Lett.* **56** (1986) 1964.
- [14] Narayan O. and Fisher D.S., *Phys. Rev. Lett.* **68** (1992) 3615; Harvard preprint (1992).
- [15] Barris A. B., *J. Phys. C* **7** (1974) 1671.
- [16] Bak P., Tang C. and Wiesenfeld K., *Phys. Rev. Lett.* **59** (1987) 381.

Note added in proof :

Following similar arguments as in the random-field case, we found that the roughness exponent of the interface for random-bond disorder is given by $\zeta \approx 0.2083 \varepsilon$, same as in the equilibrium problem discussed previously by Fisher [13]. Other critical exponents follow from equations (14) and the relation $z = 2 - (\varepsilon - \zeta)/3$, which are valid for different choices of the function A .