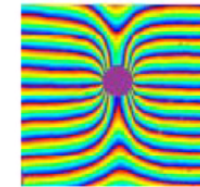


**CNRS Summer School**

**IDENTIFICATION PROCEDURES USING FULL-FIELD MEASUREMENTS APPLICATIONS IN MECHANICS OF MATERIALS AND STRUCTURES**



*May 21 –25, 2013, Montpellier*



**GDR CNRS 2519**  
**MESURES DE CHAMPS**  
**ET IDENTIFICATION EN**  
**MECANIQUE DES SOLIDES**

# **From IR thermal images to heat source assessments**

## **Part 2**

A. Chrysochoos (\*), X. Balandraud (\*\*), B. Wattrisse (\*)



3h: lecture – 4h: tutorial



(\*) Laboratoire de Mécanique et Génie Civil UMR CNRS 5508, Montpellier

(\*\*) Institut Pascal, UMR CNRS 6602, Clermont-Ferrand

# Strain-induced thermal effects

## Nature of the heat sources

- **Two « do-it-yourself » experiments**

- *break a paper clip (fingers as thermal sensor)*
- *stretch and relax a rubber band (lips as thermal sensor)*

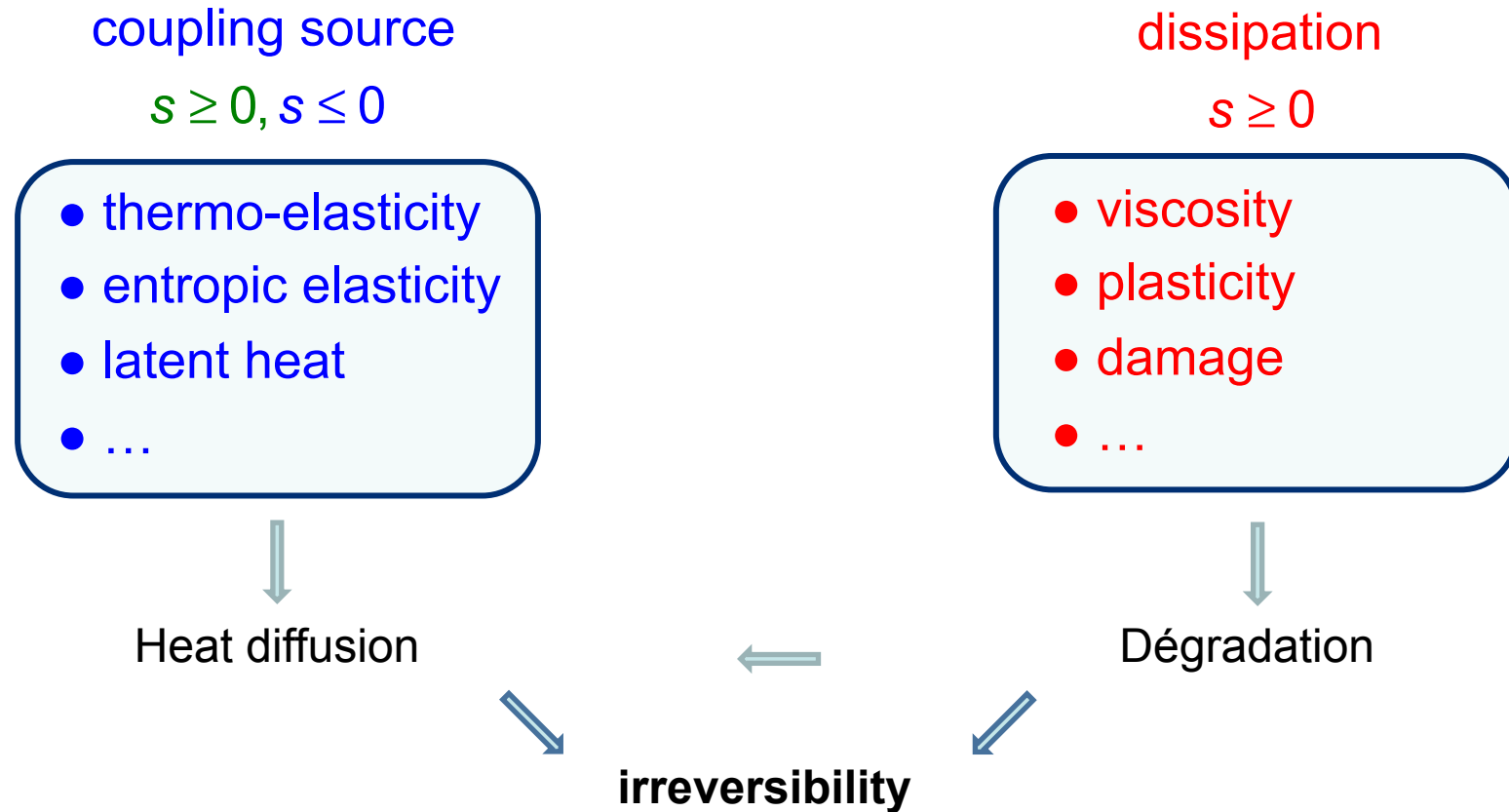
Gough 1805: “...bring the middle of the piece into slight contact with the edges of the lips...”

- **Local energy analysis of material behavior**

- *deformation energy (input)*
- *dissipation: irreversibility, material degradation*
- *coupling source: thermal sensitivity*



# What do we mean by thermo-mechanical effects?



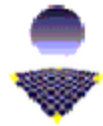
Loading-induced  $\theta \approx$  « weak » coupling  
Mechanical effects induced by loading-induced  $\theta \approx$  « strong » coupling



# Thermo-elastic effects

Gough 1805, Joule, 1857, Lord Kelvin, 1880

Micro-calorimetry techniques



100  
years  
later

...

IR thermography



**Essai cyclique (domaine "thermoélastique")**

**$R = 0$ ,  $\Delta\sigma = 360$  MPa , fréquence = 1Hz**

**Matériau : acier pour emboutissage**

Equipe Thermomécanique des Matériaux

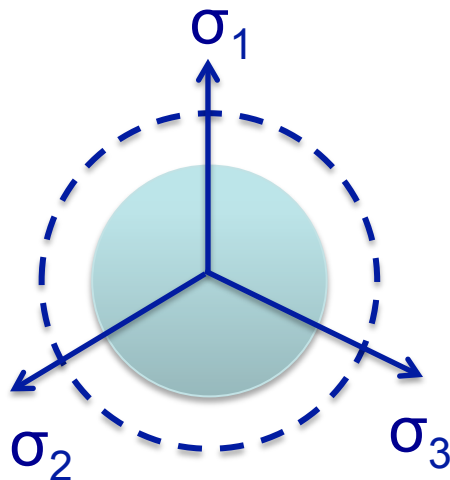
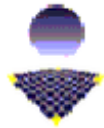
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# Thermo-elastic and plastic dissipative effects

Thermo-Elasto-Plastic material:



**Essai de traction ondulée**

$$|\dot{\sigma}| = 40 \text{ MPa.s}^{-1}$$

**Matériau : acier pour emboutissage**

Equipe Thermomécanique des Matériaux  
Laboratoire de Mécanique et de Génie Civil - UMR 5508  
Université Montpellier II



# What are the necessary investments?

## Part 1

- 1 - Thermal sciences and heat diffusion..... 4 slides
- 2 - Thermal sensors..... 14 slides
- 3 - Quantitative Infrared thermography ..... 10 slides

## Part 2

- 4 - Theoretical framework (CM + TIP) ..... 11 slides
- 5 - Image processing and heat source assessment ..... 11 slides
- 6 - Illustrative examples ..... 6 slides

## Part 3

- 7 - Tutorial session



## Balance equations

Mass:  $\dot{\rho} + \rho \operatorname{tr}(\mathbf{D}) = 0$

Linear momentum:  $\operatorname{div} \boldsymbol{\sigma} + \rho \mathbf{f} = \rho \boldsymbol{\gamma}$

Moment:  $\boldsymbol{\sigma} = \boldsymbol{\sigma}^T$

Energy:  $\rho \dot{e} = \boldsymbol{\sigma} : \mathbf{D} - \operatorname{div} \mathbf{q} + r_e$

Entropy:  $r_{si} = \rho \dot{s} - \frac{r_e}{T} + \operatorname{div} \left( \frac{\mathbf{q}}{T} \right) = \frac{d}{T} \geq 0$

Dissipation:  $d = T r_{si} \geq 0$

(via Clausius-Duhem inequality)

$\rho$  mass density  
 $\mathbf{D}$  strain rate tensor

$\boldsymbol{\sigma}$  Cauchy stress  
 $\mathbf{f}$  specific volume force  
 $\boldsymbol{\gamma}$  acceleration vector

$e$  specific internal energy  
 $\mathbf{q}$  heat influx vector  
 $r_e$  external heat supply

$r_{si}$  entropy production  
 $s$  specific entropy  
 $T$  absolute temperature  
 $d$  dissipation

***intrinsic dissipation***

$$d_1 = \boldsymbol{\sigma} : \mathbf{D} - \rho \mathbf{e}_{,\varepsilon} : \dot{\boldsymbol{\varepsilon}} - \rho \mathbf{e}_{,\alpha} \cdot \dot{\boldsymbol{\alpha}} \geq 0$$

***thermal dissipation***


$$d_2 = -\mathbf{q} / T \cdot \nabla T \geq 0$$



# Constitutive equations

Generalized standard material formalism

[Son & Halphen, 75]

|                                |   |   |
|--------------------------------|---|---|
| state variables                | $\{T, \varepsilon, \alpha\}$  |  SPH |
| internal/free energy potential | $e(s, \varepsilon, \alpha)$ $\psi(T, \varepsilon, \alpha)$  |   |
| state equations                | $-s = \psi_{,T}$ $\sigma^r = \rho \psi_{,\varepsilon}$ $\mathbf{A} = \rho \psi_{,\alpha}$                                 |   |
| dissipation potential          | $\varphi(q, \dot{\varepsilon}, \dot{\alpha}; T, \dots)$   |   |
| evolution equations            | $-\frac{\nabla T}{T} = \varphi_{,q}$ $\sigma^{ir} = \varphi_{,\dot{\varepsilon}}$ $-\mathbf{A} = \varphi_{,\dot{\alpha}}$ |   |

**irreversibility**

material degradation

$$d_1 = \sigma^{ir} : \dot{\varepsilon} - A \cdot \dot{\alpha}$$

heat diffusion

$$d_2 = \varphi_{,q} \cdot q = -\frac{\nabla T}{T} \cdot q$$





## Energy balance (I)

$$\begin{aligned} W_{\text{def}}^{\bullet} &= \sigma : \dot{\varepsilon} = \sigma^r : \dot{\varepsilon} + \sigma^{\text{ir}} : \dot{\varepsilon} \\ &= \underbrace{\sigma^r : \dot{\varepsilon} + A_{\alpha} \cdot \dot{\alpha}}_{W_e^{\bullet} + W_s^{\bullet}} + d_1 \end{aligned}$$

$W_e^{\bullet}$  : elastic energy rate

$d_1$  : intrinsic dissipation

$W_s^{\bullet}$  : stored energy rate



... incomplete balance!!



## Energy balance (II)

- internal energy rate

$$\rho \dot{e} = \rho C \dot{T} + (\sigma^r : \dot{\epsilon} + A \cdot \dot{\alpha}) - (T \sigma_{,T}^r : \dot{\epsilon} + T A_{,T} \cdot \dot{\alpha})$$

$$= \rho C \dot{T} + w_e \dot{} + w_s \dot{} - w_{thc} \dot{} \quad \left( \underbrace{\quad\quad\quad}_{\text{heat diffusion}} \right)$$

- heat diffusion equation

$$\rho C \dot{T} + \text{div} \mathbf{q} = \underbrace{\sigma^{ir} : \dot{\epsilon} - \mathbf{A} \cdot \dot{\alpha}}_{d_1} + T \sigma_{,T}^r : \dot{\epsilon} + T \mathbf{A}_{,T} \cdot \dot{\alpha} + r_e$$

C.1:  $C$  specific heat

C.2:  $\mathbf{q} = -\mathbf{k} : \text{grad} T$

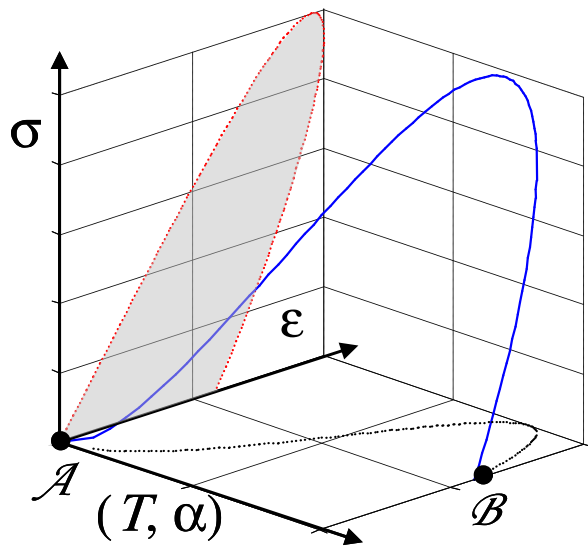


DIC required

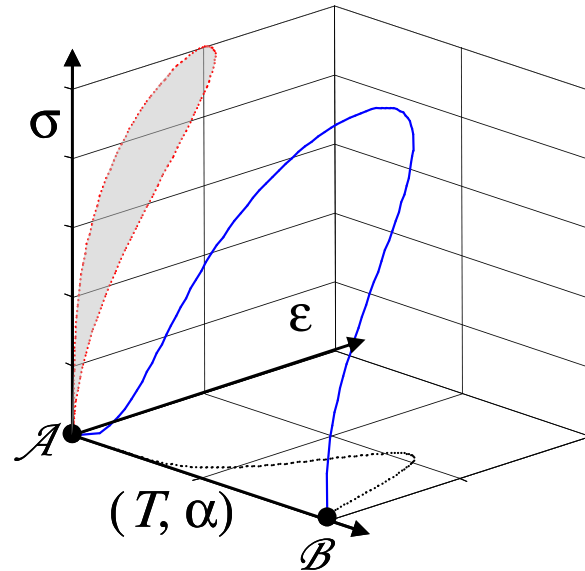
$$C.3: \dot{T} = \frac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla T$$



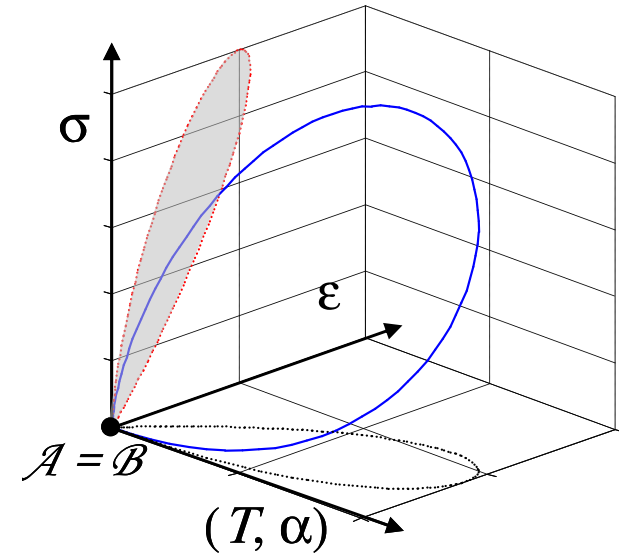
## Energy balance (III)



(i)  $\mathcal{A} \neq \mathcal{B}$



(ii)  $\varepsilon_{\mathcal{A}} = \varepsilon_{\mathcal{B}}$



(iii)  $\mathcal{A} = \mathcal{B}$

$$(i) \quad w_{\text{def}} = \int_{t_{\mathcal{A}}}^{t_{\mathcal{B}}} \sigma : \dot{\varepsilon} dt = \int_{t_{\mathcal{A}}}^{t_{\mathcal{B}}} d_1 dt + \int_{t_{\mathcal{A}}}^{t_{\mathcal{B}}} (\rho \dot{e} - \rho C \dot{T} + w_{\text{thc}}) dt$$

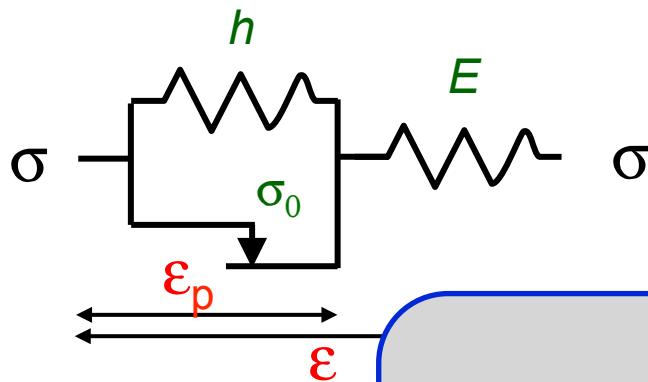
(ii) mechanical hysteresis loop :  $w_{\text{def}} = A_h$  if ... and  $A_h$  induced by ... 

(iii) load-unload cycle = thermodynamic cycle  $w_{\text{def}} = \int_{t_{\mathcal{A}}}^{t_{\mathcal{B}}} d_1 dt + \int_{t_{\mathcal{A}}}^{t_{\mathcal{B}}} w_{\text{thc}} dt$

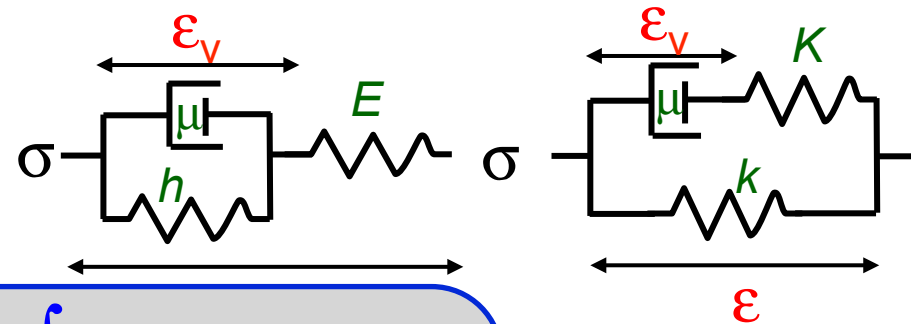


# Energy balance over a cycle

Prager  
linear kinematic hardening



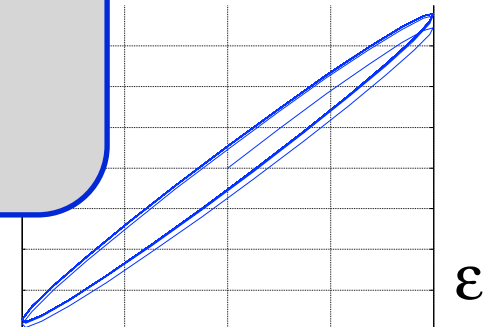
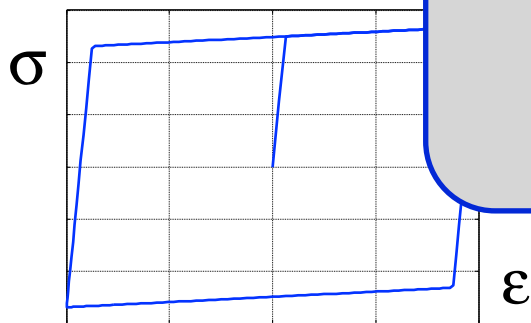
Poynting-Thomson (or Zener)  
linear viscoelasticity



$$A_h = \int_C \sigma \dot{\epsilon} d\tau$$

$$w_d = \int_C \sigma_0 |\dot{\epsilon}_p| d\tau \text{ or } \int_C \mu |\dot{\epsilon}_v|^2 d\tau$$

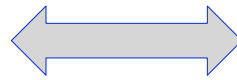
$$A_h = w_d$$



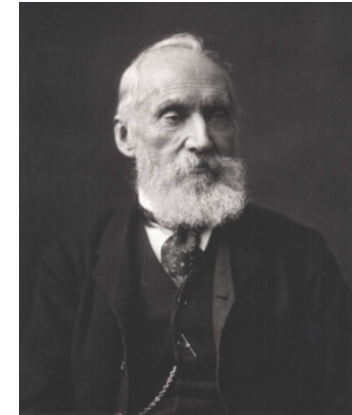
# Energy balance over a cycle

The simplest linearised thermoelastic model

$$\begin{cases} \varepsilon = \frac{\sigma}{E} + \lambda_{th}(T - T_0) \\ \dot{T} + \frac{T - T_0}{\tau_{th}} = -\frac{E\lambda_{th}T\dot{\varepsilon}}{\rho C} \end{cases}$$



« 0D » approach  
linear heat losses



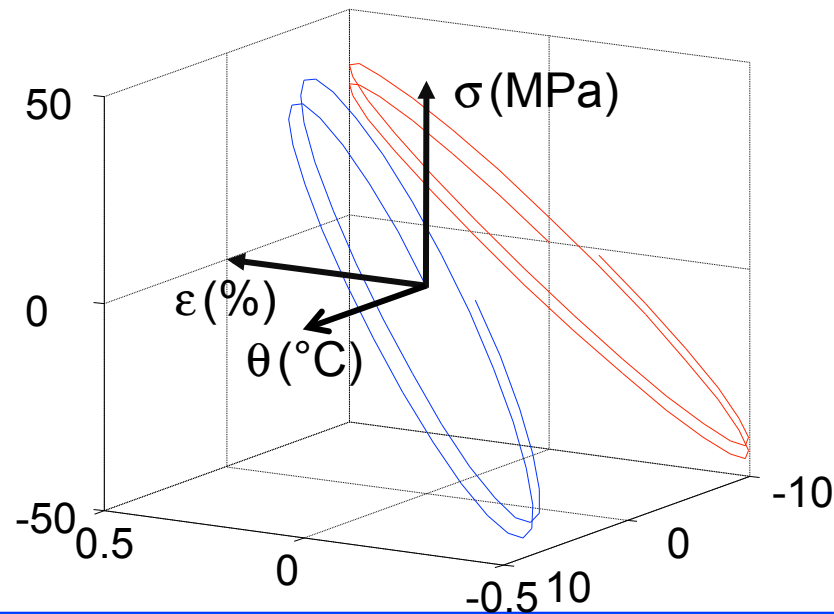
William Thomson  
Lord Kelvin (1824-1907)

$E = 1000 \text{ MPa}$   
 $\rho = 1000 \text{ kg.m}^{-3}$   
 $C = 1000 \text{ J.kg}^{-1}.\text{K}^{-1}$

$\lambda_{th} = 50 \cdot 10^{-5} \text{ K}^{-1} \times 100$

$\tau_{th} = 30 \text{ s}$

$T_0 = 294 \text{ K}$

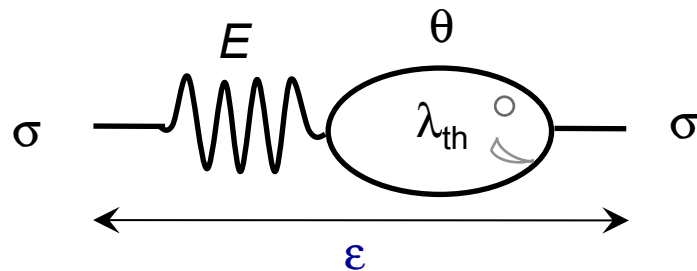


Towards a  
stabilized  
cycle ...



# Dissipative vs. coupling effects

Thermo-elastic coupling (i.e.  $d_1=0$ )  
**Strong coupling**



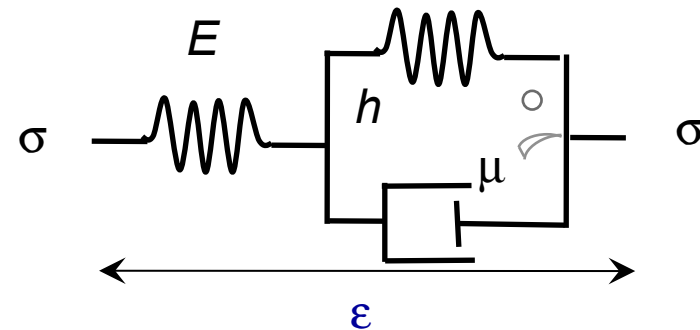
State variables ( $\theta, \epsilon$ )

$$\begin{cases} \epsilon = \frac{\sigma}{E} + \lambda_{th} \theta \\ \dot{\theta} + \frac{\theta}{\tau_{th}} = -\frac{E\lambda_{th}(T_0 + \theta)\dot{\epsilon}}{\rho C} \end{cases}$$

Rheological equation

$$\sigma + \tau_{th} \dot{\sigma} = E\epsilon + E\tau_{th} \left( 1 + \frac{E\lambda_{th}^2 T_0}{\rho C} \right) \dot{\epsilon}$$

Dissipative viscous process  
**Weak coupling**



State variables ( $\epsilon, \epsilon_v$ )

$$\begin{cases} \sigma = E(\epsilon - \epsilon_v) \\ \sigma = h\epsilon_v + \mu \dot{\epsilon}_v \end{cases}$$

Rheological equation

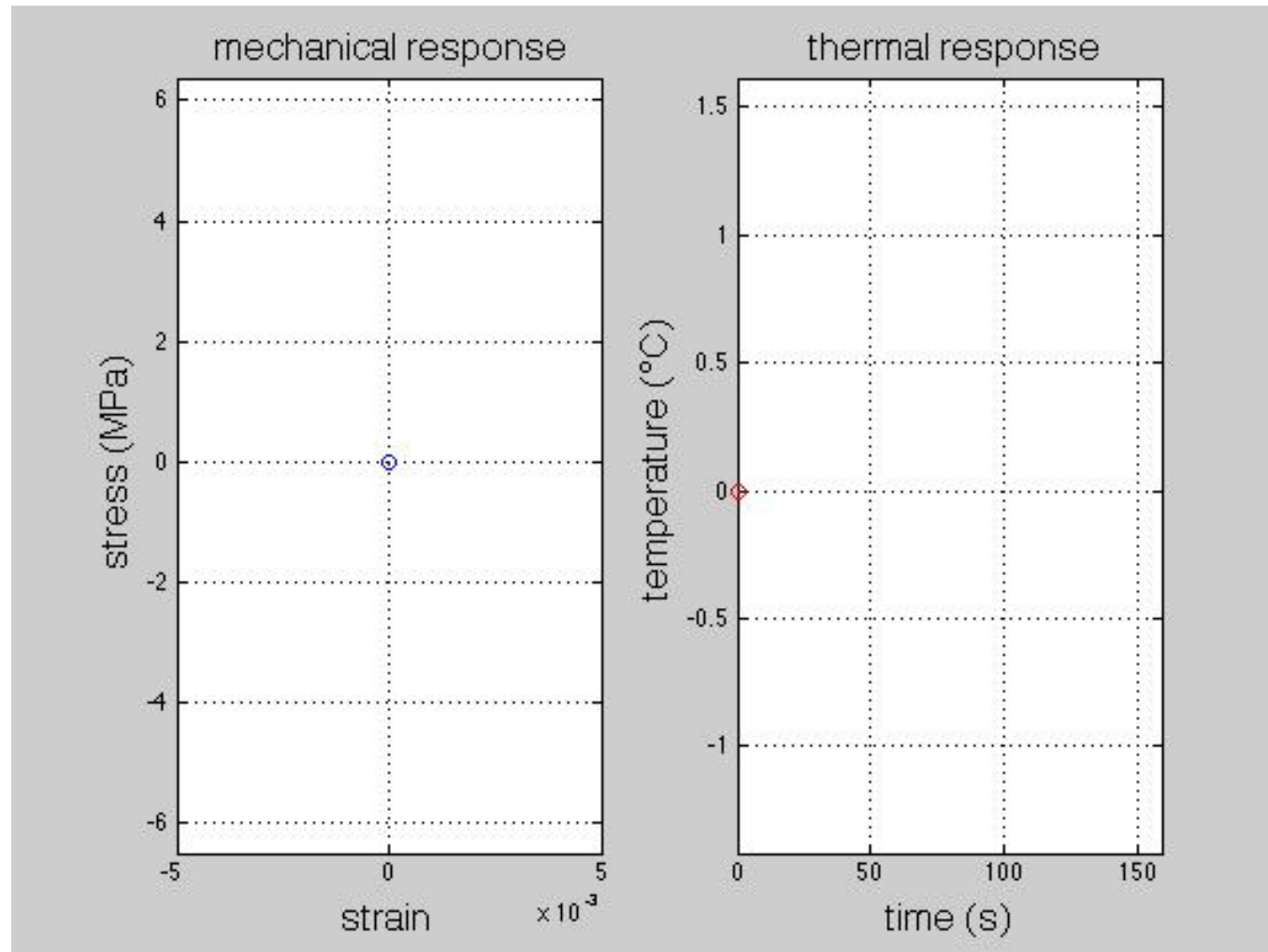
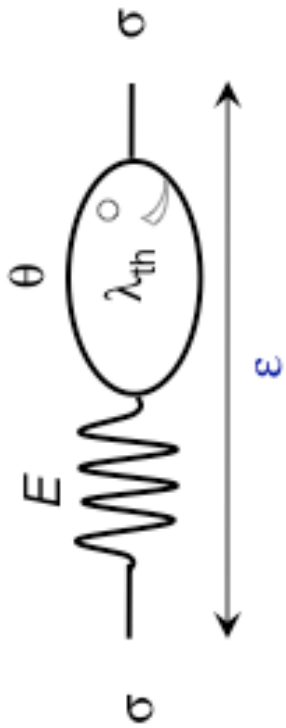
$$\sigma + \frac{\mu}{E+h} \dot{\sigma} = \frac{Eh}{E+h} \epsilon + \frac{E\mu}{E+h} \dot{\epsilon}$$



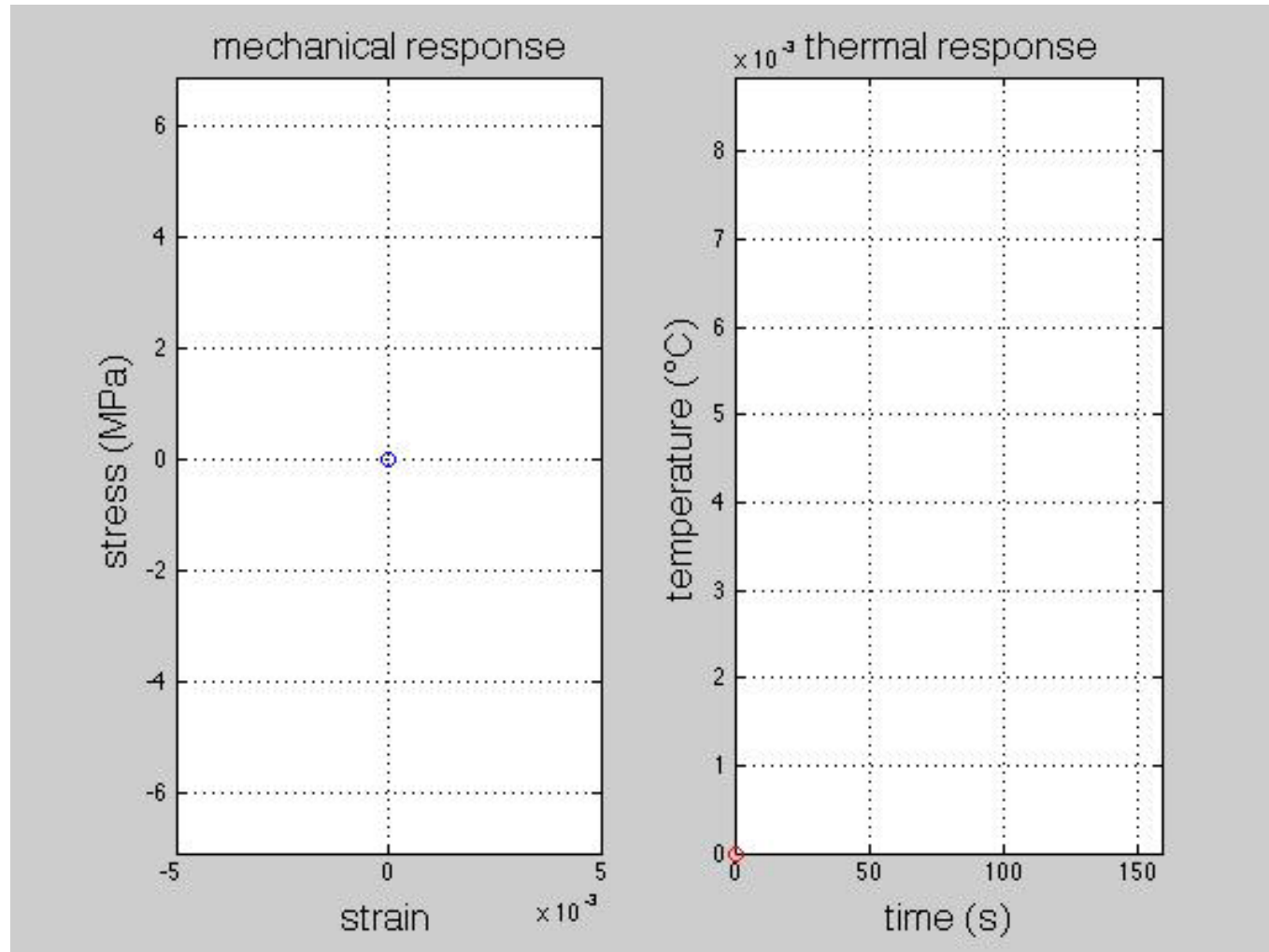
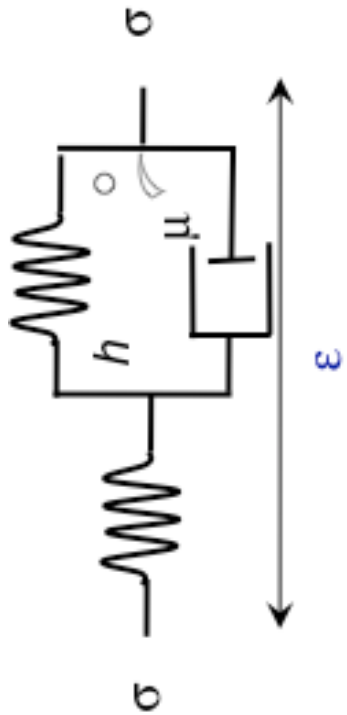
Viscoanalysis of polymers



## Thermoelastic effects : 'strong' coupling



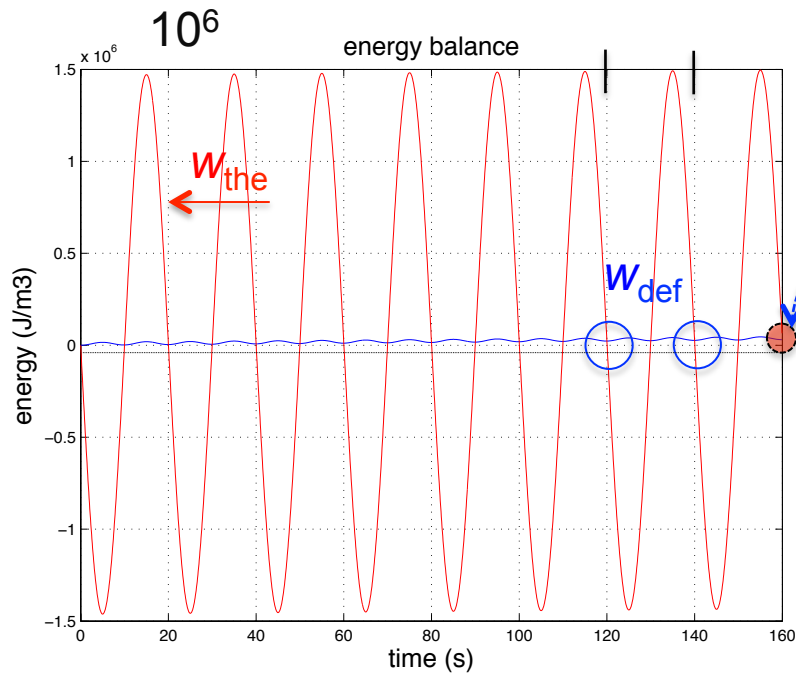
## Viscous dissipation : 'weak' coupling





# Energy balance: comparative study

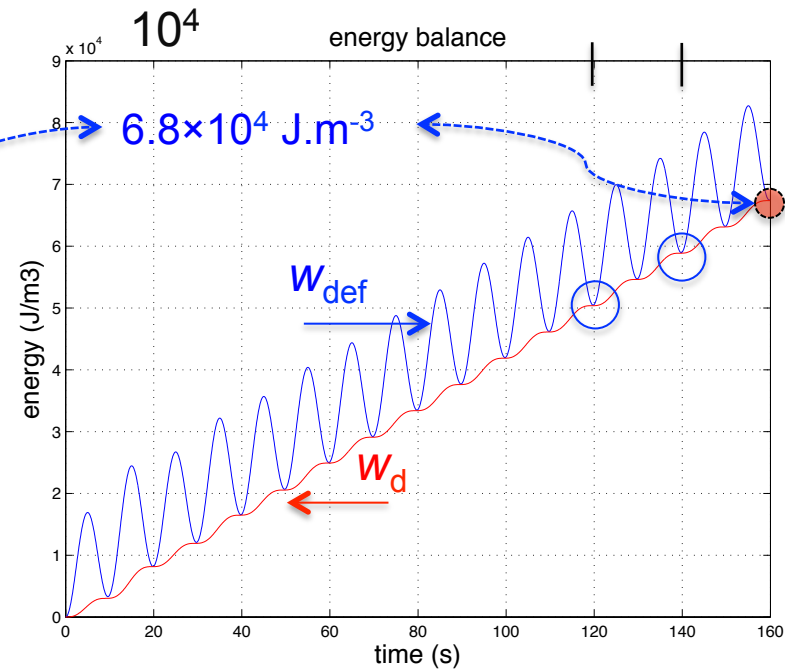
## Thermelasticty



$$\Delta W_{the} \gg \Delta W_{def}$$

Range

## Viscoelasticity



$$W_d \leq W_{def}$$

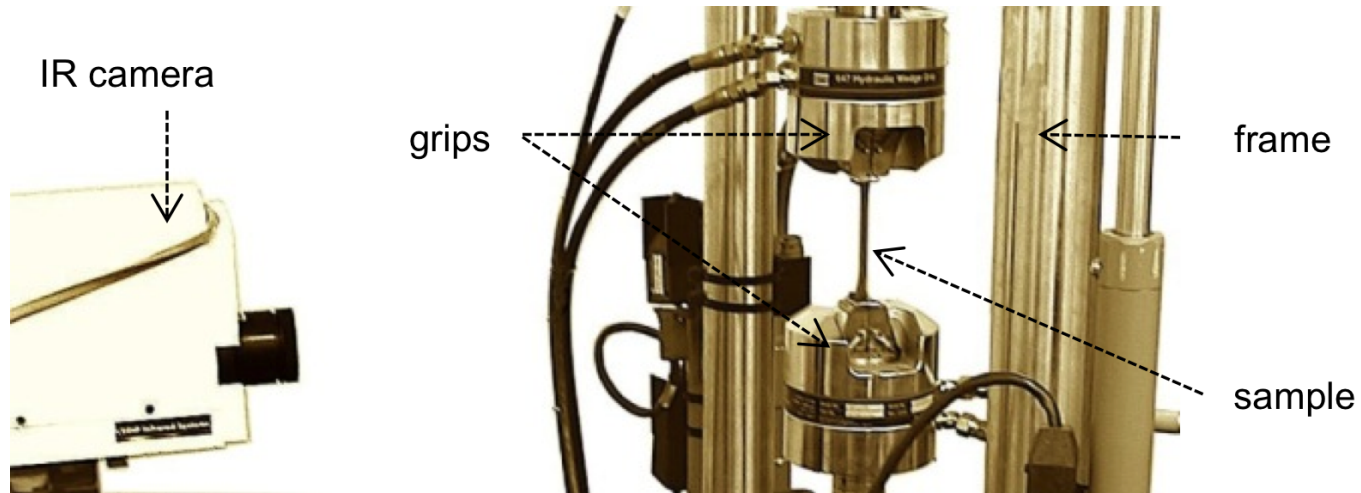
Energy balance at the end of each thermodynamic cycle

$$W_{the} = W_{def}$$

$$W_d = W_{def}$$

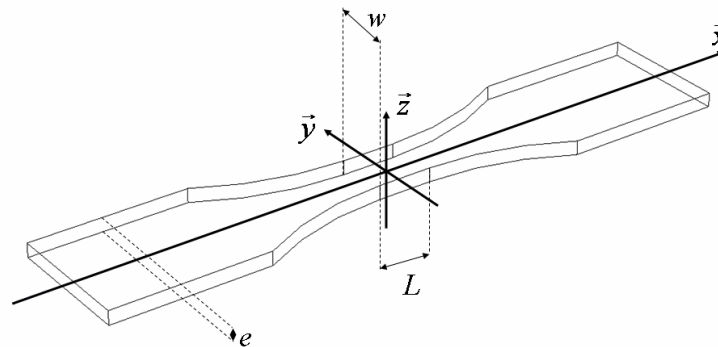


# Experimental context



Thin flat samples

$e$ : thickness  
 $w(x)$ : width  
 $L$ : length



## Experimental context

- Thanks to heat diffusion:

$$\bar{\theta}(t, x, y) = \frac{1}{e} \int_{-e/2}^{e/2} \theta(t, x, y, z) dz \approx \theta_{IR}(t, x, y)$$

- Integrating the 3D heat equation over the thickness leads to:

$$\rho C \left( \frac{\partial \bar{\theta}}{\partial t} + \frac{\bar{\theta}}{\tau_{th}} \right) - k \Delta_{x,y} \bar{\theta} = \bar{w}_h$$

|                                  |
|----------------------------------|
| (un)known<br>measured<br>unknown |
|----------------------------------|

... with linear Fourier boundary conditions (  $\tau_{th}$  )

- To assess heat sources, it is « only » necessary to estimate **partial differential operators** using **discrete noisy data fields**.
- When sources are known (e.g. zero), then **thermophysical parameters** can be identified (cf tutorials on diffusivity identification... )



Here, heat source assessment = direct PDO estimate !

$$\rho C \left( \underbrace{\frac{\partial \bar{\theta}}{\partial t}}_2 + \underbrace{\frac{\bar{\theta}}{\tau_{th}}}_1 \right) - k \underbrace{\Delta_{x,y} \bar{\theta}}_3 = \bar{w}_h$$



1 : out-of plane heat losses



2 : stored/released heat rate



3 : 2D laplacian, in-plane heat losses

- Several possible methods, but ....:
  - *finite differences*: ... beware of thermal noise!!
  - *spectral method*: truncated projection onto the eigenfunction basis
  - *Fourier analysis*: DFT + filtering (cf. tutorial 'convolution filtering')
  - *LSQ fitting*: local space-time approximation
  - ...
  - interpolation, splines, ... beware of thermal noise!!



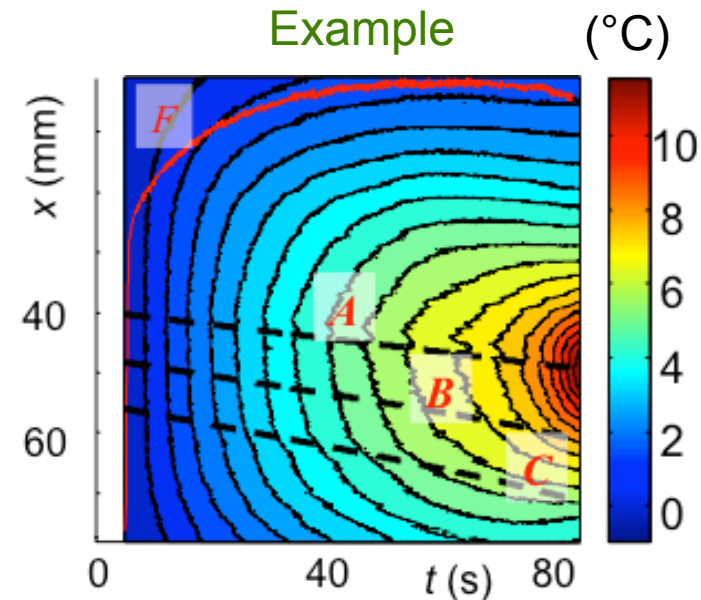
## Simplified but convenient thermal models

**1D Model:** integration over a cross-section, linear heat exchanges

$$\rho C \left( \dot{\bar{\theta}} + \frac{\bar{\theta}}{\tau_{th}^{1D}} \right) - k \Delta_x \bar{\theta} = \bar{w}_h \dot{\phantom{\theta}}$$

$\bar{\theta}(t,x) = \bar{\theta}_{IR}(t,x)$  where  $\bar{\theta}_{IR}$  is now a width-averaged thermopprofile

$$\bar{\theta}_{jk} = \frac{1}{(M-m)} \sum_{i=m}^M \theta_{ijk}^{IR}$$



**0D local model:** homogeneous distribution of sources, use of spectral solution for linear, homogeneous B.C. (heat diffusion = differential problem)

$$\rho C \left( \dot{\theta} + \frac{\theta}{\tau_{eq}} \right) = w_h \dot{\phantom{\theta}}$$

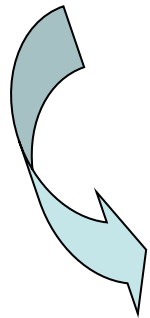


# Discrete noisy thermal data

Thermal noise:

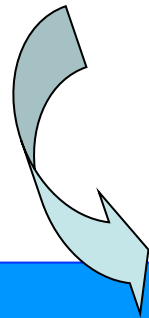
- uniform power spectrum
- Gaussian probability distribution

Heat source assessment

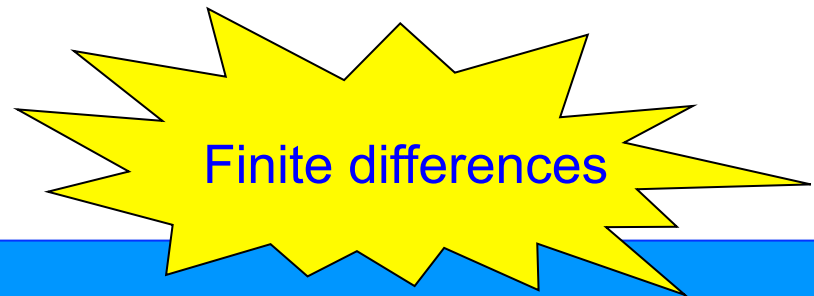
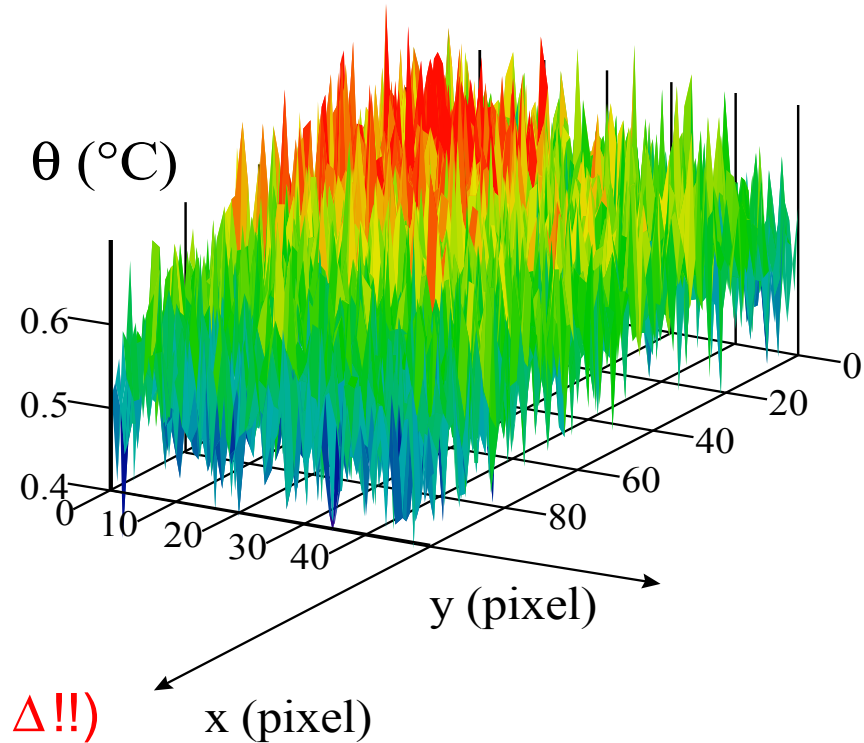


$$\frac{d}{dt}, \Delta_{x,y}$$

Partial derivatives:  
Noise amplifier (principally  $\Delta$ !!)



Example



# Use of truncated spectral solution

Use of the eigenfunctions (EF) and eigenvalues (ev) of the Laplacian operator  $-\Delta_{x,y}$  :

knowledge of EF  $f_k(x)$ ,  $g_l(y)$  and ev  $\lambda_k$  et  $\mu_l$

$$\theta(x, y, t) = \sum_{k=1}^{\infty} \sum_{l=1}^{\infty} a_{kl}(t) f_k(x) g_l(y)$$

with:  $-\Delta_x f_k(x) = \lambda_k f_k(x)$   
 $-\Delta_y g_l(y) = \mu_l g_l(y)$

$$\Delta_{xy} \theta_{ap}(x, y) = \sum_{k=1}^{n_k} \sum_{l=1}^{n_l} a_{kl}(t) (\lambda_k + \mu_l) f_k(x) g_l(y)$$

Analytical Laplacian computation !!



$$\theta_{ap}(x, y) = \sum_{k=1}^{n_k} \sum_{l=1}^{n_l} a_{kl}(t) f_k(x) g_l(y)$$

$\theta_{ap}$  approximation de  $\theta_{IR}$   
 $a_{kl}$  projection  $\theta_{IR}$  of on to  $h_{kl}(x, y) = f_k(x) \times g_l(y)$

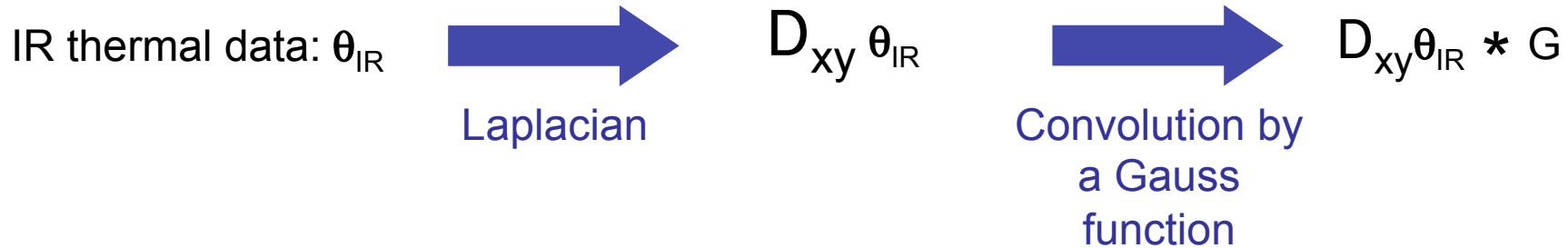
## tricky issues :

- numerical-analytical computation of the ev and EF,
- numerical computation of the projections ( $a_{kl}$ ):  
 accurate numerical integration time-consuming

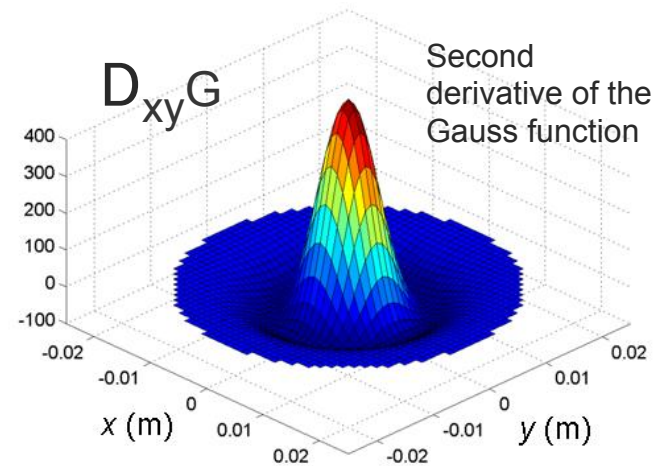


# Derivative Gaussian Filter

(Cf. Tutorials)



$\xrightarrow{\text{Convolution and differentiation commute}}$   $\theta_{IR} * D_{xy} G$   
Filtering and differentiation in a same operation



## tricky issues :

- choice of the size of the kernel
- choice of the standard deviation of the Gaussian function

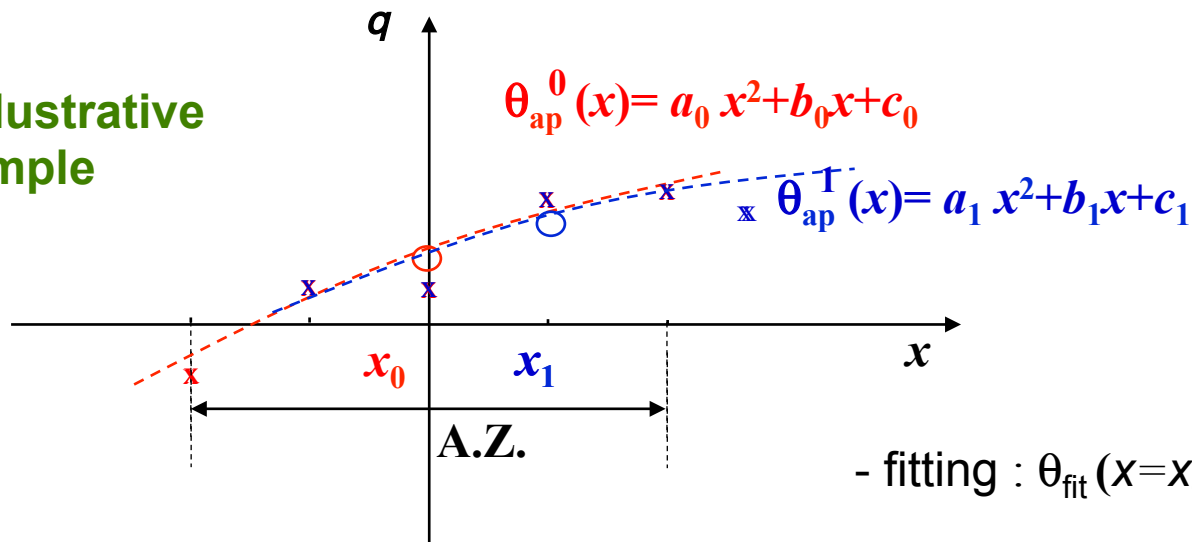




# Local LSQ fitting

Local space-time approximation of the temperature field:

1D illustrative example



- fitting :  $\theta_{\text{fit}}(x=x_0) = a_0 x_0^2 + b_0 x_0 + c_0$

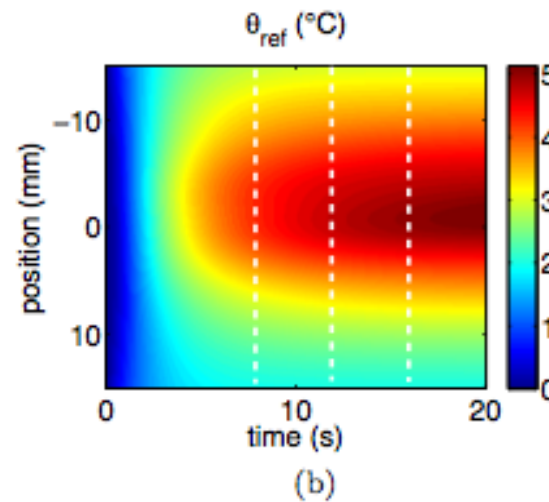
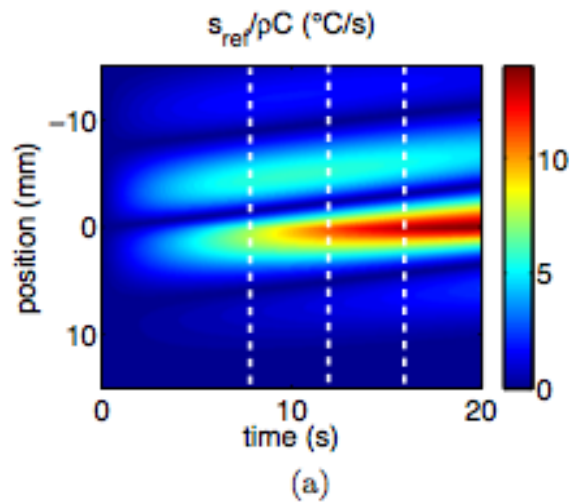
- differentiation :  $\Delta_x \theta_{\text{fit}}(x=x_0) = 2 a_0$

**tricky issues** : choice of fitting parameters  
type of approximation function  
approximation zone size

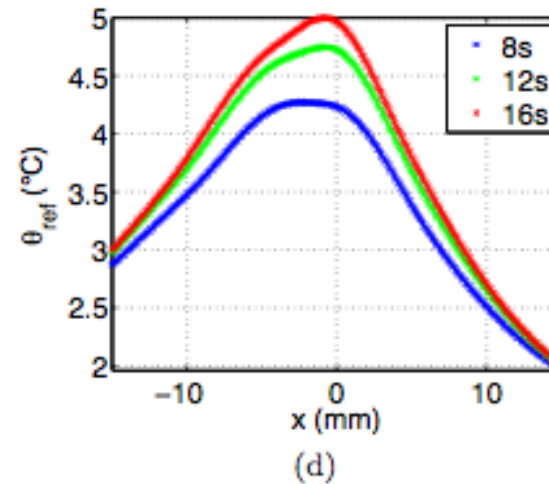
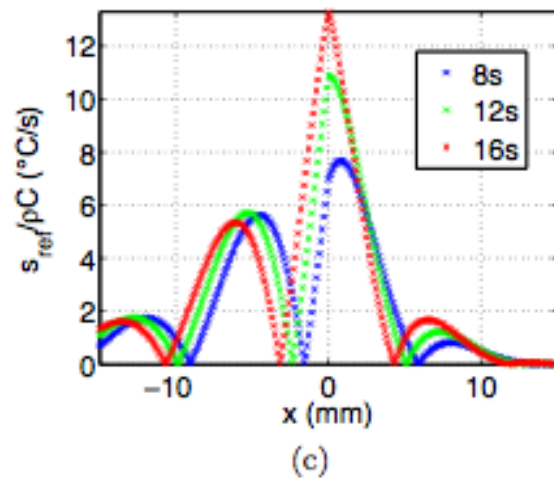


# Comparative analysis

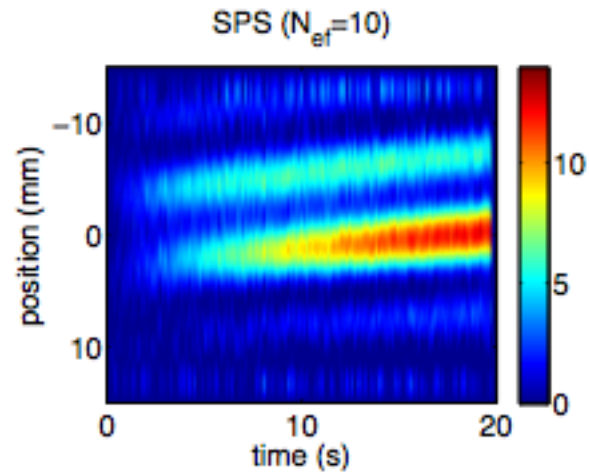
1D benchmark test: high diffusivity coefficient, boundary conditions, noise ...



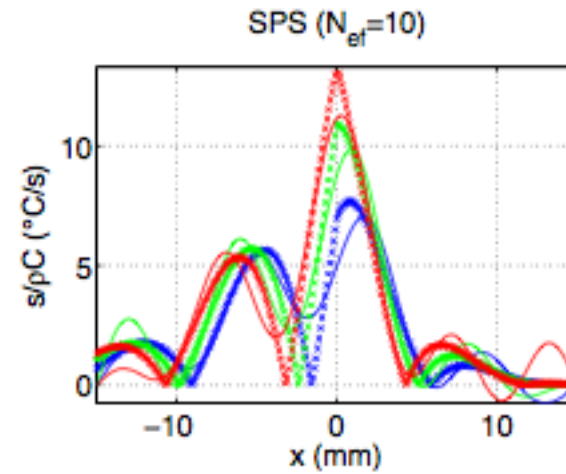
$k=360 \text{ W.m}^{-1}.\text{K}^{-1}$   
 $\rho = 8920 \text{ kg.m}^{-3}$   
 $C = 385 \text{ J.kg}^{-1}.\text{K}^{-1}$   
 $D \approx 10^{-4} \text{ m}^2.\text{s}^{-1}$   
NETD= 150 mK  
T.B.C.: unsteady



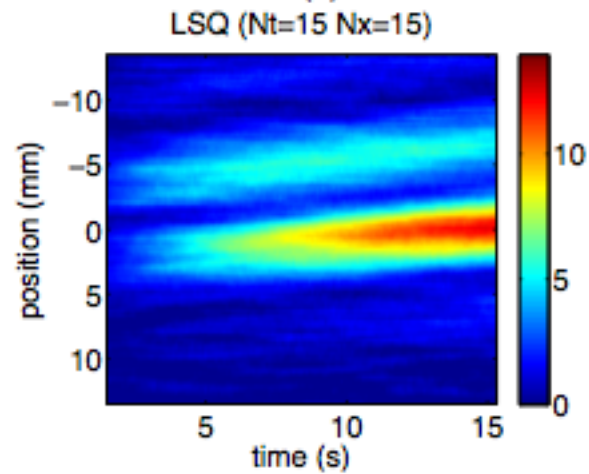
# Comparative analysis



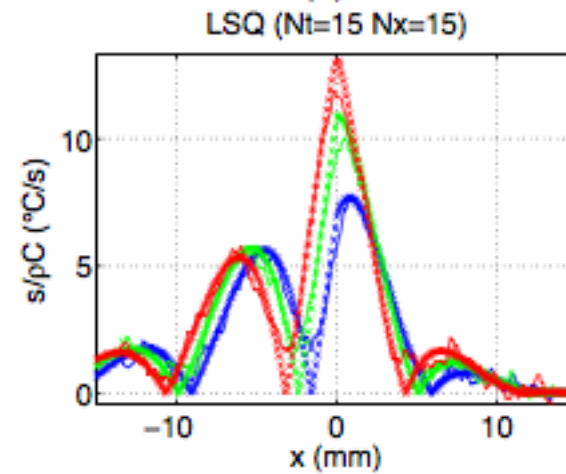
(a)



(b)



(c)



(d)

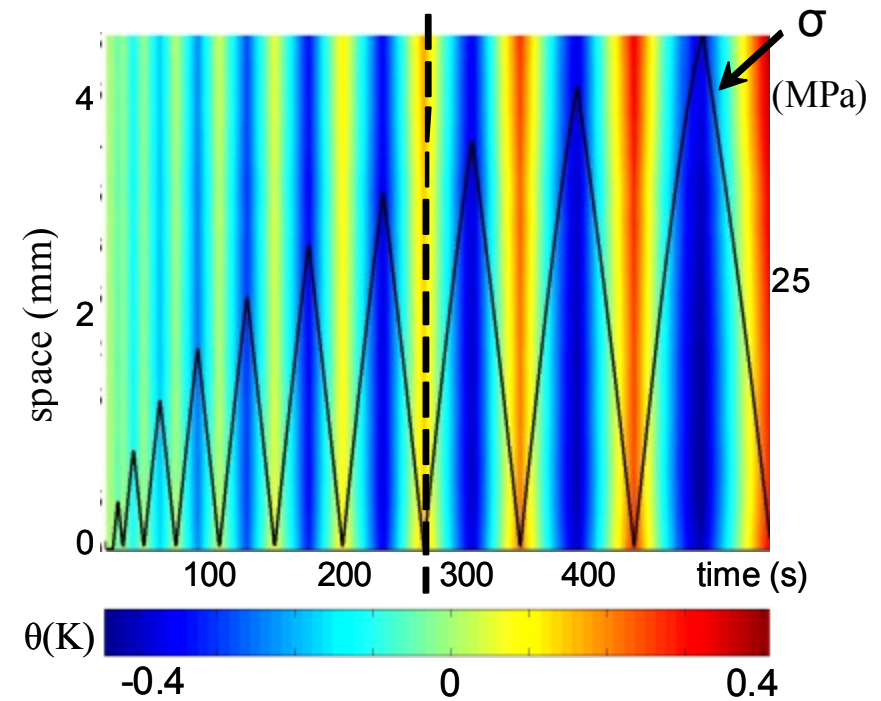
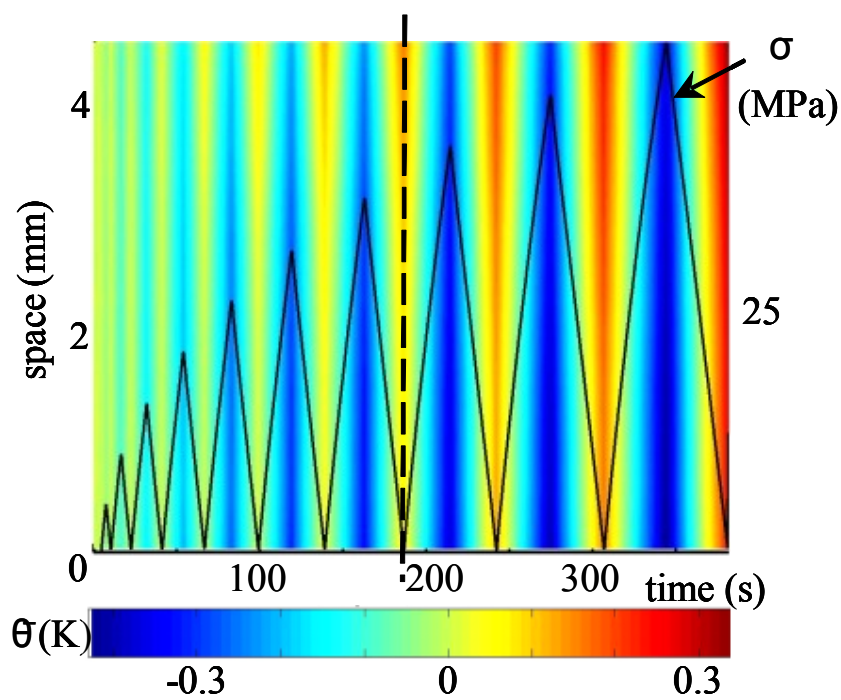


# PMMA & PC : polymers in glassy state

*isothermal SPH : the realm of linear viscoelasticity*

J. Alfrey (48), M. Biot (65), F. Sidoroff (70-75)

*Equivalence of series and parallel models (e.g. P.T. and Z. models)*



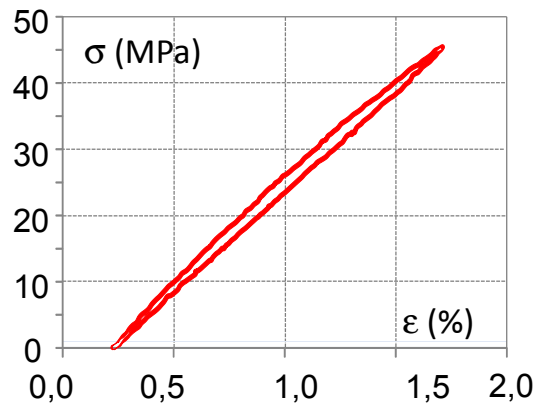
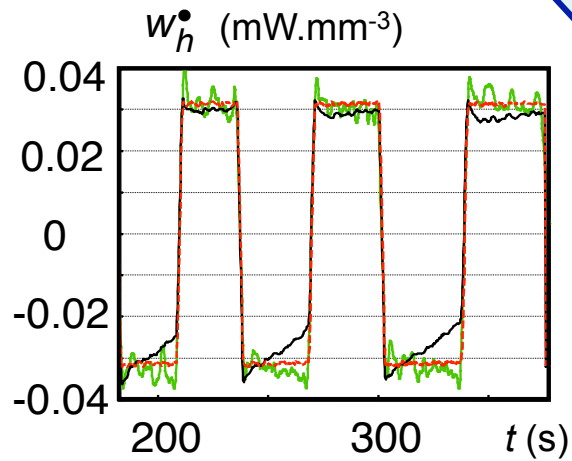
[S. Moreau, PhD 03]

Non isothermal process  
Introduction of thm couplings ....

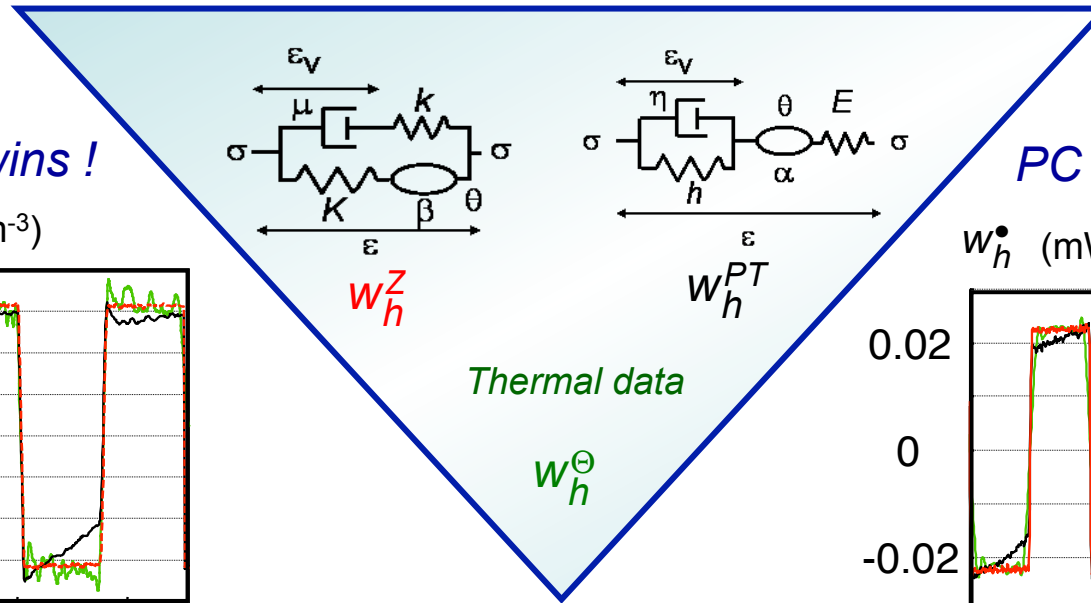
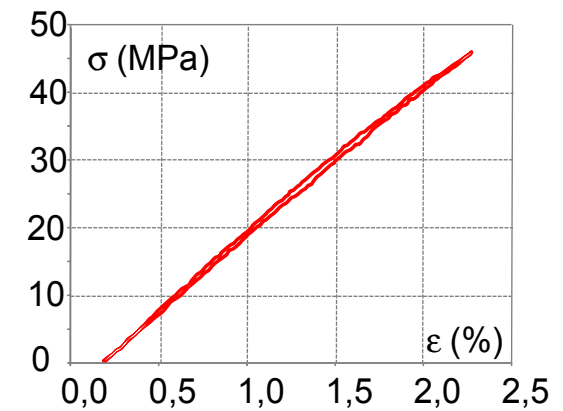
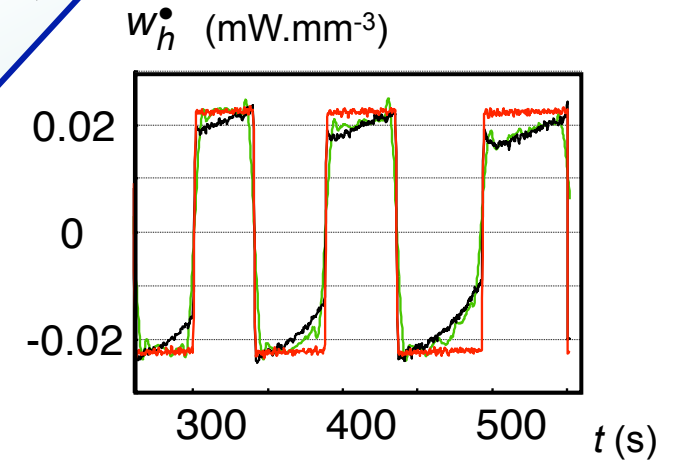


# PMMA & PC : polymers in glassy state

PMMA : Zener wins !



PC : Poynting wins !



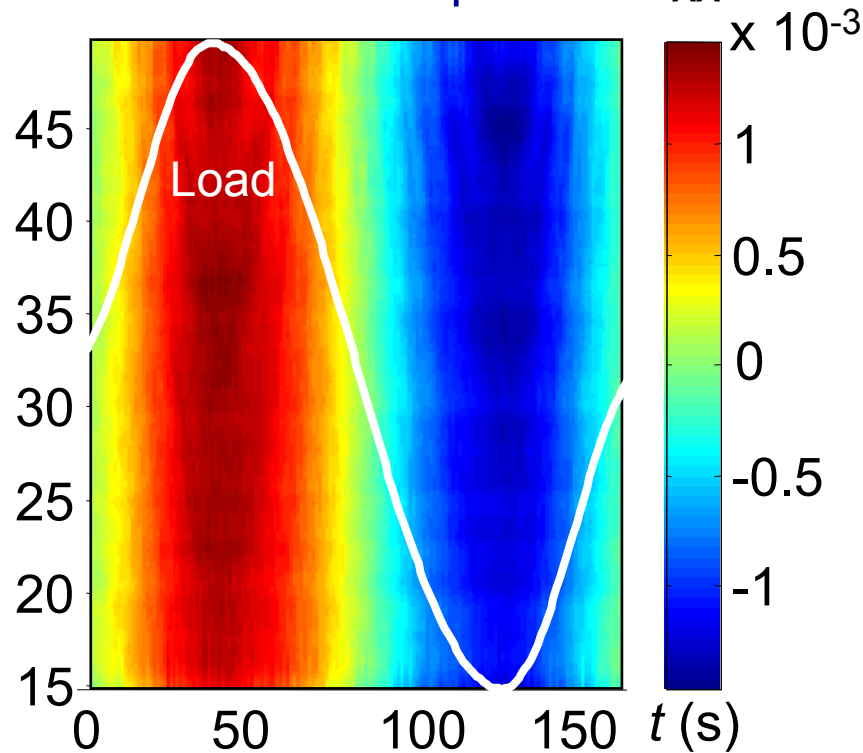
$A_h \approx w_{the}$



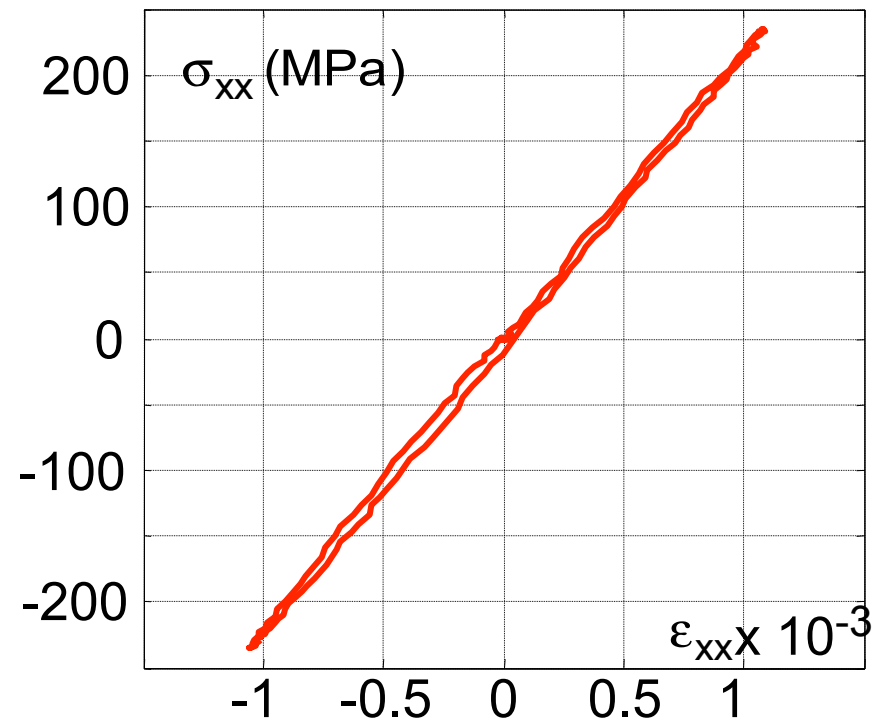
# High cycle fatigue : mechanical response

DP 600 steel  $\Delta\sigma = 471$  MPa,  $R_\sigma = -1$ ,  $f_L = 5.5 \cdot 10^{-3}$  Hz

Tensile Strain profile  $\varepsilon_{xx}(t, x)$



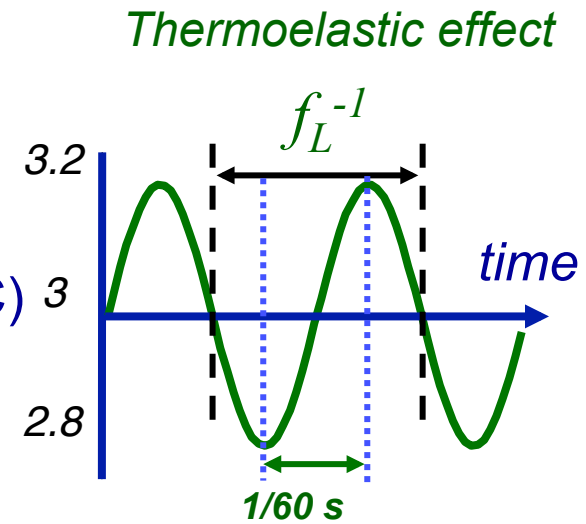
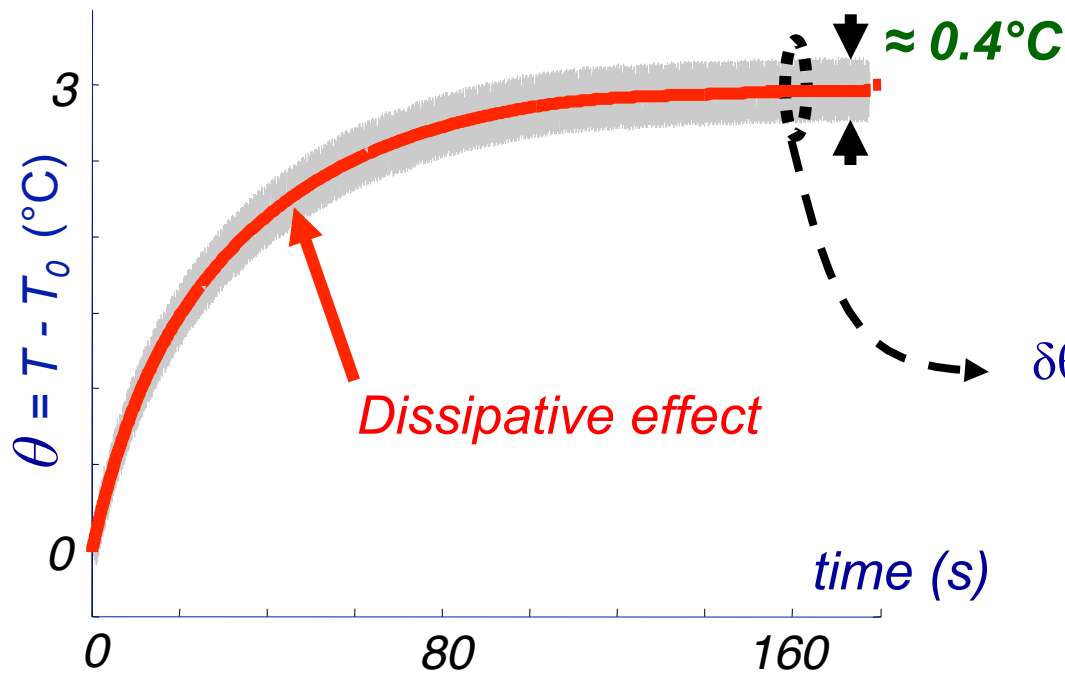
Hysteresis loop



# High cycle fatigue : thermal response

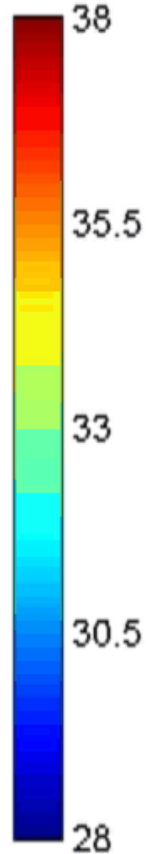
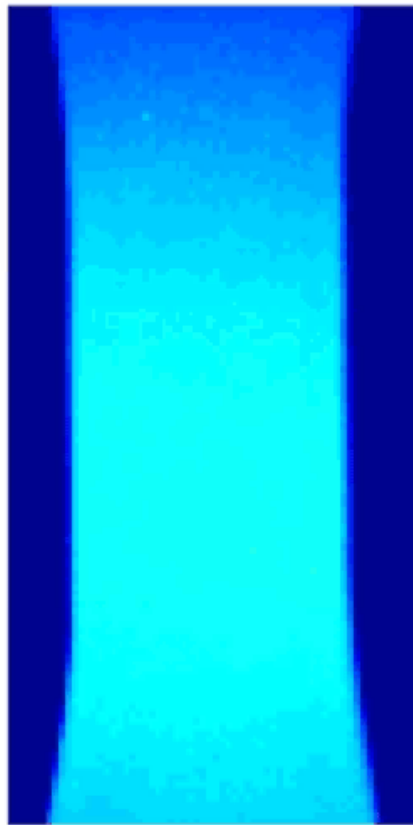
HCF test on DP 600 steel

$\Delta\sigma = 500$  MPa,  $R_\sigma = \sigma_{min}/\sigma_{max} = -1$  et  $f_L = 30$  Hz

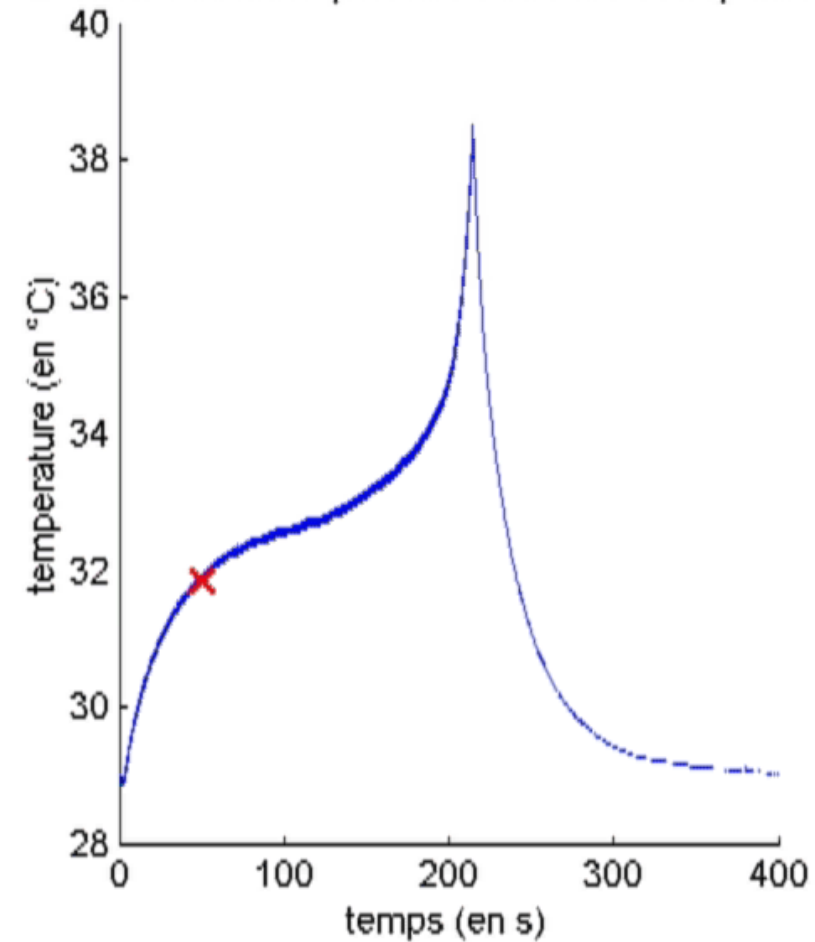


## High cycle fatigue : temperature fields

Image n° 1000 (en °C)



Evolution de la température au centre de l'éprouvette



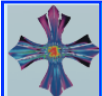
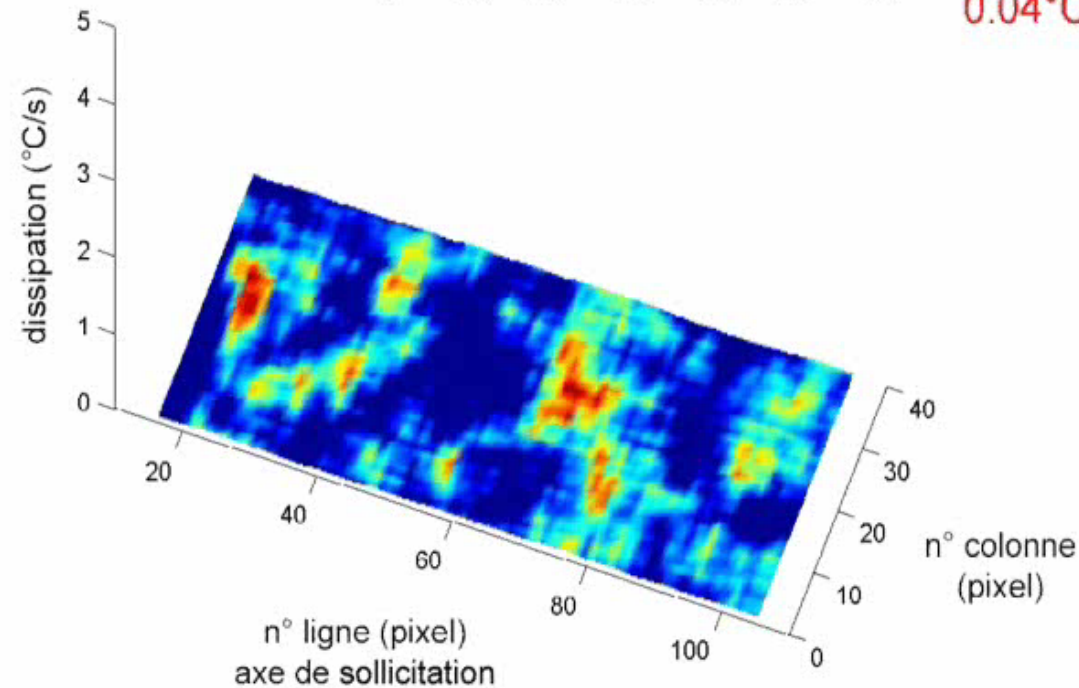
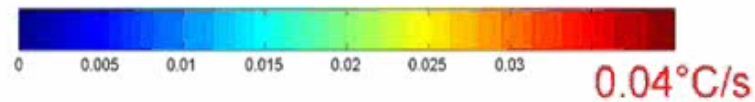


# High cycle fatigue : dissipation fields

|  |                |
|--|----------------|
| Amplitude de contrainte $\Delta\sigma/2$ | <b>133 MPa</b> |
| Rapport de charge                        | $R_o = 0$      |

|                                       |                         |                               |                |
|---------------------------------------|-------------------------|-------------------------------|----------------|
| Dimension de la partie utile (h*1*te) | 10x9x2.5mm <sup>3</sup> | Nombre de cycles par blocs    | 20000          |
| Fréquence de sollicitation Fs         | 50 Hz                   | Fréquence d'acquisition Fa    | 19 Hz          |
| Temps du bloc                         | 400s                    | Résolution spatiale           | 0.196 mm/pixel |
| Lieu de fissuration                   | Pixel 98x6              | ligne limite de la zone utile | 35 à 86        |

0 cycles 20000



# Conclusion and prospects

Theoretical objective: energy analysis of material behavior

- *Heat source assessment* → *energy balance* → *constitutive equations*
- *Stored energy* ↔ *state eq.* ; *dissipation* ↔ *evolution eq.*
- *Full field measurements: initial and boundary conditions*

IR metrology:

- *Thermal stability, pixel calibration*
- *Frame rate, image lost*
- *Parasitic radiation ...*

Heat source identification:

- *Thermo-physical parameters ; thermal noise ; heat diffusion*
- *Direct methods : 0-2D approaches* → *real 3D inverse problem*
- *Use of POD-SVD pre-processing*

