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Turbulence modelling in cavitating flows

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LABORATOIRE DES ECOULEMENTS GEOPHYSIQUES ET INDUSTRIELS

Outline

1. General considerations
2. One-fluid RANS equations
3. Limitation of eddy viscosity
4. Compressibility terms
5. Wall models
6. Improved models - hybrid RANS/LES
7. One-fluid filtered equations and LES
8. Conclusions

General considerations

A large quantity of opened questions

Questions

- Kolmogorov spectrum, slope $\neq$ -5/3
- Kolmogorov scale / size of two-phase structures.
- Induced turbulence or pseudo-turbulence.
- Compressibility effects on turbulence.
- Anisotropy of the Reynolds tensor.
- Increase or decrease of the turbulence intensity.
- Cavitation-turbulence interaction at small scales.

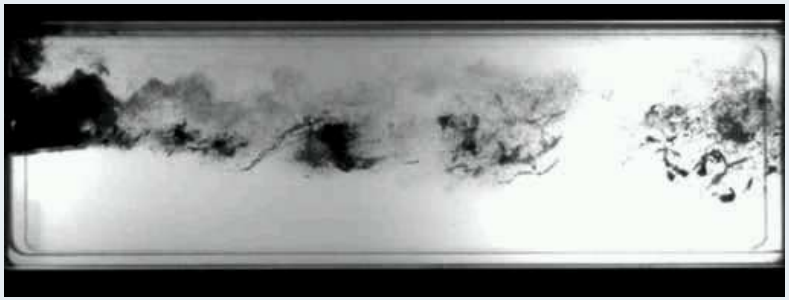
Remarks

- no DNS data or turbulent quantities measurements.
- non universality of two-phase flows.

Example of flows

Cavitation pockets

Cavitation in vortex



Multiple scales

Turbulence scales

- DNS : all scales are resolved
- LES : large scale are resolved and small scales are modelled
- RANS : mean flow is resolved and turbulence is modelled
- Hybrid RANS/LES : model adapts to the mesh resolution

Phase scales

- DNS : resolution of each fluid + interface
- pseudo-DNS : resolution of each fluid + tracking of the interface (VoF, level set...)
- filtered (LES) and averaged (RANS) models :
  - two-fluid model,
  - reduced models,
  - one-fluid model

Averaged one-fluid equations (RANS)

- Temporal averaged or ensemble averaged equations
- The same operator for two-phase structure and turbulent structure
- Boussinesq analogy used similarly to single-phase flows
- Transport-equation models : $k - \varepsilon$, $k - \omega$,...

Filtered one-fluid equations (LES)

- First works in incompressible flow without phase transition
- Lots of sub-grid terms - problem of modelling
- Link with the DNS

Averaged equations

Temporal phase average

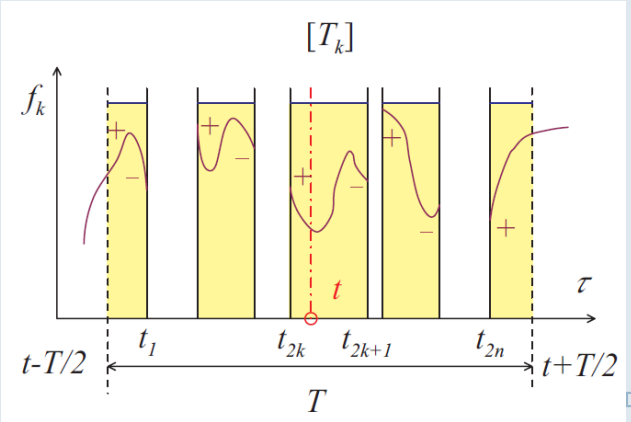
Averaging over a time T :

$$\overline{\phi}_k = \frac{1}{\mathrm{T}} \int_{\mathrm{T}} \phi_k(x,\tau)\, d\tau$$

Averaging over the time T_k of presence of the phase k :

$$\overline{\overline{\phi}}_k = \frac{1}{\mathrm{T}_k} \int_{\mathrm{T}_k} \phi_k(x,\tau)\, d\tau$$

Void fraction : $\alpha(x,t) = \dfrac{\mathrm{T}_k}{\mathrm{T}} = \dfrac{\overline{\phi}_k}{\overline{\overline{\phi}}_k}$



Mass-weighted average (Favre average)

For a variable $\rho\phi$, a mass-weighted average :

$$\widetilde{\phi_k} = \frac{\overline{\rho_k \phi_k}}{\overline{\rho_k}} = \frac{\overline{\overline{\rho_k \phi_k}}}{\overline{\overline{\rho_k}}} \qquad \text{and} \qquad \tilde{\phi} = \frac{\sum \alpha_k \overline{\overline{\rho_k}} \widetilde{\phi_k}}{\sum \alpha_k \overline{\overline{\rho_k}}} = \frac{\sum \overline{\rho_k} \widetilde{\phi_k}}{\sum \overline{\rho_k}}$$

Mass-weighted decomposition

An average part and a fluctuation (two-phase and turbulent contribution) :

$$\begin{aligned} \rho_k &= \overline{\overline{\rho_k}} + \rho'_k, & u_k &= \widetilde{u_k} + u''_k, & \phi_k &= \widetilde{\phi_k} + \phi''_k \\ \overline{\overline{\rho'_k}} &= 0 \quad , & \overline{\overline{\rho_k u''_k}} &= 0 \quad , & \overline{\overline{\rho_k \phi''_k}} &= 0 \end{aligned}$$

Conservative equations for the phase k

Mass conservation :

$$\frac{\partial \alpha_k \overline{\overline{\rho}}_k}{\partial t} + \operatorname{div}\left(\alpha_k \overline{\overline{\rho}}_k \widetilde{u}_k\right) = \Gamma_k$$

Momentum conservation :

$$\frac{\partial \alpha_k \overline{\overline{\rho_k u_k}}}{\partial t} + \operatorname{div}\left(\alpha_k \overline{\overline{\rho}}_k \widetilde{u}_k \otimes \widetilde{u}_k + P_k\right) = \operatorname{div}\left(\alpha_k (\overline{\overline{\tau}}_k + \tau_k^t)\right) + \mathbb{M}_k$$

Total energy conservation :

$$\frac{\partial \alpha_k \overline{\overline{\rho}}_k \left(\widetilde{E}_k + k_k\right)}{\partial t} + \operatorname{div}\left(\alpha_k \rho_k (\widetilde{E}_k + k_k) \widetilde{u}_k\right) = \operatorname{div}\left(\alpha_k P_k \widetilde{u}_k\right) + \operatorname{div}\left(\alpha_k (\overline{\overline{\tau}}_k + \tau_k^t) \widetilde{u}_k\right) - \operatorname{div}\left(\alpha_k (\overline{\overline{q}}_k + q_k^t)\right) + \mathbb{Q}_k$$

New unknowns : Reynolds stress tensor τ_k^t and turbulent heat flux q_k^t

Fluctuating fields

The velocity fluctuating field is not divergence free (even with incompressible phases) :

$$\frac{\partial u'_{k,l}}{\partial x_l} = -\frac{\partial \overline{\overline{u}}_{k,l}}{\partial x_l} = \frac{1}{\alpha_k} u'_k . n_k \delta_l$$

It leads to supplementary unknowns, called "compressible terms".

Equation for the momentum fluctuation :

$$\begin{aligned}
\rho_k \frac{\partial u''_{k,i}}{\partial t} + \rho_k \widetilde{u}_{k,l} \frac{\partial u''_{k,i}}{\partial x_l} \quad + \quad & \rho_k u''_{k,l} \frac{\partial \widetilde{u}_{k,i}}{\partial x_l} + \rho_k u''_{k,l} \frac{\partial u''_{k,i}}{\partial t} \\
= \quad & \frac{\partial \mathbb{T}_{k,il}}{\partial x_l} + \rho_k \widetilde{u}_{k,i} \frac{\partial \widetilde{u}_{k,l}}{\partial x_l} \\
& + \frac{\rho_k}{\overline{\overline{\rho}}_k \alpha_k} \left[-\frac{\partial \alpha_k \overline{\overline{\mathbb{T}}}_{k,il}}{\partial x_l} - \mathbb{M}_k + \frac{\partial}{\partial x_l} \left(\alpha_k \overline{\overline{\rho}}_k \widetilde{u''_{k,i} u''_{k,l}} \right) \right] \\
& + \frac{\rho_k}{\overline{\overline{\rho}}_k \alpha_k} \left[\widetilde{u}_{k,i} \left(\frac{\partial \alpha_k \overline{\overline{\rho}}_k}{\partial t} + \widetilde{u}_{k,l} \frac{\partial \alpha_k \overline{\overline{\rho}}_k}{\partial x_l} \right) \right]
\end{aligned}$$

Turbulent kinetic energy equation for the phase k

it is possible to write the equation for the phasic Reynolds stress and the phasic TKE k_k

$$\frac{\partial \alpha_k \overline{\overline{\rho}}_k k_k}{\partial t} + \mathrm{div}\left(\alpha_k \overline{\overline{\rho}}_k \, k_k \widetilde{u}_k\right) = \alpha_k \overline{\overline{\rho}}_k \left(P_k - \varepsilon_k + \Pi_k + M_k + D_k\right) + \Gamma_k K^{\Gamma}$$

with :

$$\overline{\overline{\rho}}_k \, \alpha_k P_k = -\alpha_k \overline{\overline{\rho}}_k \, \widetilde{u''_{k,i} u''_{k,l}} \frac{\partial \widetilde{u}_{k,i}}{\partial x_l} \qquad \text{Production term}$$

$$\overline{\overline{\rho}}_k \, \alpha_k \epsilon_k = \alpha_k \overline{\overline{\tau'_{k,il} \frac{\partial u''_{k,i}}{\partial x_l}}} \qquad \text{Dissipation rate}$$

$$\overline{\overline{\rho}}_k \, \alpha_k \Pi_k = \alpha_k \overline{\overline{p'_k \frac{\partial u''_{k,i}}{\partial x_i}}} \qquad \text{Pressure-dilatation term}$$

$$\overline{\overline{\rho}}_k \, \alpha_k M_k = \alpha_k \overline{\overline{u''_{k,i}}} \left[-\frac{\partial \overline{\overline{p}}_k}{\partial x_i} + \frac{\partial \overline{\overline{\sigma}}_{k,il}}{\partial x_l} \right] \qquad \text{Mass flux term}$$

$$\overline{\overline{\rho}}_k \, \alpha_k D_k = -\frac{\partial}{\partial x_l} \left[\alpha_k \overline{\overline{P'_k u''_{k,i}}} \, \delta_{il} - \alpha_k \overline{\overline{\tau'_{k,il} u''_{k,i}}} \right] - \frac{\partial \alpha_k \rho_k \overline{\overline{\frac{u''_{k,i} u''_{k,i}}{2} u''_{k,l}}}}{\partial x_l} \qquad \text{Diffusion term}$$

$\Gamma_k K^{\Gamma}$ Mass transfer term

Homogeneous mixture equations or one-fluid model (1)

Mixture mean quantities

- mixture density and presure : $\rho_m = \sum \alpha_k \overline{\overline{\rho}}_k$, $P_m = \sum \alpha_k \overline{\overline{P}}_k$
- mixture internal energy : $\rho_m e_m = \sum \alpha_k \overline{\overline{\rho}}_k \widetilde{e}_k$
- mass center velocity : $\rho_m u_{m,i} = \sum \alpha_k \overline{\overline{\rho}}_k \, \widetilde{u}_{k,i}$
- mixture viscosity : $\mu_m = \sum \alpha_k \overline{\overline{\mu}}_k$
- mixture viscous stress tensor : $\tau_{m,ij} = \sum \alpha_k \overline{\overline{\tau_{k,ij}}}$
- mixture heat flux : $q_m = \sum \alpha_k \overline{\overline{q_k}}$

Mixture turbulent quantities

- mixture turbulent kinetic energy : $k_m = \sum \alpha_k k_k = \sum \alpha_k \widetilde{u''^2_{k,i}}/2$
- mixture Reynolds stress tensor : $\tau^t_{m,ij} = -\sum \alpha_k \overline{\overline{\rho_k u''_{k,i} u''_{k,j}}}$
- mixture eddy viscosity : $\mu_{tm} = \sum \alpha_k \mu_{tk}$
- mixture turbulent heat flux : $q^t_m = \sum \alpha_k q^t_k$
- mixture dissipation rate : ε_m, mixture specific dissipation ω_m...

Conservative equations

Mass equation :

$$\frac{\partial \rho_m}{\partial t} + \operatorname{div}(\rho_m u_m) = 0$$

Momentum equation :

$$\frac{\partial \rho_m u_m}{\partial t} + \operatorname{div}(\rho_m\, u_m \otimes u_m + P_m) = \operatorname{div}(\tau_m + \tau_m^t)$$

Energy equation :

$$\frac{\partial \rho_m (E_m + k_m)}{\partial t} + \operatorname{div}(\rho_m (E_m + k_m) u_m) = \operatorname{div}(-P_m u_m) - \operatorname{div}(q_m - q_m^t) + \operatorname{div}\left[(\tau_m + \tau_m^t) u_m\right]$$

Mixture turbulent kinetic energy equation

$$\frac{\partial \rho_m k_m}{\partial t} + \mathrm{div}\left(\rho_m k_m u_m\right) = \rho_m P_m + \rho_m \Pi_m - \rho_m \epsilon_m + \rho_m M_m + \rho_m D_m + \Gamma_m K_m^{\Gamma}$$

It is assumed that each phase shares the same fluctuating velocity $u_i^{''}$, the same fluctuating pressure $P^{'}$ and the same fluctuating viscous stress $\tau_{ij}^{'}$:

$$\rho_m P_m = \tau_{m,il}^{t} \frac{\partial u_{m,i}}{\partial x_l} \quad ; \quad \rho_m \Pi_m = \overline{P^{'} \frac{\partial u_i^{''}}{\partial x_i}}$$

$$\rho_m \epsilon_m = \overline{\tau_{il}^{'} \frac{\partial u_i^{''}}{\partial x_l}} \quad ; \quad \Gamma_m K_m^{\Gamma} = 0$$

$$\rho_m M_m = \overline{u_i^{''}} \left[-\frac{\partial p_m}{\partial x_i} \right] + \overline{u_i^{''}} \left[\frac{\partial \sigma_{m,il}}{\partial x_l} - \sum_k \overline{\overline{\sigma}}_{k,il} \frac{\partial \alpha_k}{\partial x_l} \right]$$

$$\rho_m D_m = -\frac{\partial}{\partial x_l} \left[\overline{p^{'} u_i^{''}} \delta_{il} - \overline{\sigma_{il}^{'} u_i^{''}} \right] - \frac{\partial \rho_m \widetilde{\frac{u_i^{''} u_i^{''}}{2}} u_l^{''}}{\partial x_l}$$

Main features

- A fluctuation is associated to a mixture quantity !
- The fluctuating velocity field is not divergence free $\rightarrow$ supplementary terms, difficult to model.
- The pressure-dilatation term $\rho_m \Pi_m$ is null in mean but not instantaneously.
- The mixture dissipation ε is not only solenoidal.
  Inhomogeneous and dilatational (or compressible) contributions.

$$\rho\epsilon \approx \underbrace{2\mu\,\overline{\omega'_{ik}\omega'_{ik}}}_{\rho\varepsilon_s} + \underbrace{2\,\mu\,\frac{\partial}{\partial x_k}\left[\frac{\partial \overline{u'_k u'_l}}{\partial x_l} - 2\,\overline{u'_k s'_{ll}}\right]}_{\rho\varepsilon_{inh}} + \underbrace{\frac{4}{3}\,\mu\,\overline{s'_{kk}s'_{ll}}}_{\rho\varepsilon_d}$$

- The diffusion term is modelled with a gradient formulation.
- The mixture dissipation equation ε is completely modelled following the single-phase formulation.

Boussinesq assumption

Boussinesq analogy and mixture eddy viscosity assumption μ_{tm} :

$$\tau^t_{m,ij} = \mu_{tm}\left[\frac{\partial u_{m,i}}{\partial x_j} + \frac{\partial u_{m,j}}{\partial x_i} - \frac{2}{3}\,\mathrm{div}\,u_m\delta_{ij}\right] - \frac{2}{3}\rho_m k_m \delta_{ij}$$

Evaluation of the mixture eddy viscosity with transport-equation models.

Turbulent Fourier law

Fourier law analogy and mixture thermal conductivity λ^t_m :

$$q^t_m = -\lambda^t_m\,\mathrm{grad}\,T_m$$

Assumption of constant turbulent Prandtl number Pr_t :

$$\lambda^t_m = \sum \alpha_k \lambda^t_k = \sum \alpha_k \frac{\mu^t_k C_{pk}}{Pr^t} \qquad \text{approximed by} \quad \lambda^t_m \simeq \frac{\mu^t_m C_{pm}}{Pr^t}$$

Usual model, $k-\varepsilon$ model for a mixture

transport-equation models equivalent to single-phase turbulent models.
all supplementary terms are neglected.
only the solenoidal dissipation is taken into account.

$$\frac{\partial \rho\, k}{\partial t} + \mathrm{div}\left[\rho\, u\, k - \left(\mu + \frac{\mu_t}{\sigma_k}\right) \mathrm{grad}\, k\right] = \rho P_k - \rho\varepsilon$$

$$\frac{\partial \rho\varepsilon}{\partial t} + \mathrm{div}\left[\rho\, u\, \varepsilon - \left(\mu + \frac{\mu_t}{\sigma_\varepsilon}\right) \mathrm{grad}\, \varepsilon\right] = c_{\varepsilon 1}\frac{\varepsilon}{k}\rho P_k - \rho c_{\varepsilon 2} f_2 \frac{\varepsilon^2}{k}$$

$$\mu_t \approx \rho \frac{k^2}{\varepsilon_s}$$

Remarks

- a large quantity of assumptions.
- introduction of wall treatment (damping functions or wall functions)
- a large quantity of problems !

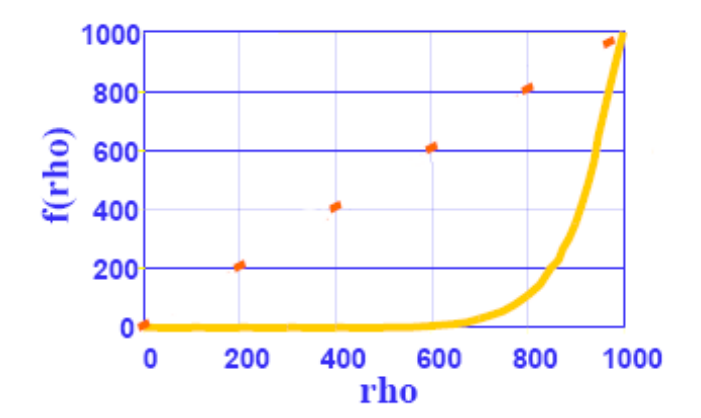
Limitation of the eddy viscosity

The Reboud correction

$$\mu_t = f(\rho) C_\mu \frac{k^2}{\epsilon}$$

$$f(\rho) = \rho_v + \left(\frac{\rho_v - \rho}{\rho_v - \rho_l}\right)^n (\rho_l - \rho_v)$$

n is usually set to 10



SST correction - Bradshaw's assumption for 2D boundary layer

$$\mu_t = \frac{\rho k/\omega}{\max\left(1, \frac{\Omega F_2}{a_1 \omega}\right)} \; ; \; a_1 = 0.3 \; ; \; \Omega = \sqrt{2\Omega_{ij}\Omega_{ij}} \text{ with } \Omega_{ij} = \frac{1}{2}\left(\frac{\partial \tilde{u}_i}{\partial x_l} - \frac{\partial \tilde{u}_l}{\partial x_i}\right)$$

Realisability constrains of Durbin

$$\mu_t = \min\left(C_\mu^0 \, ; \, \frac{c}{s\sqrt{3}}\right) \frac{\rho k^2}{\epsilon} \; ; \; 0 \leq c \leq 1 \; ; \; C_\mu^0 = 0.09 \; ; \; s = \frac{k}{\epsilon} S \text{ with } S = 2 S_{ij} S_{ij} - \frac{2}{3} S_{kk}^2$$

The Venturi 4°

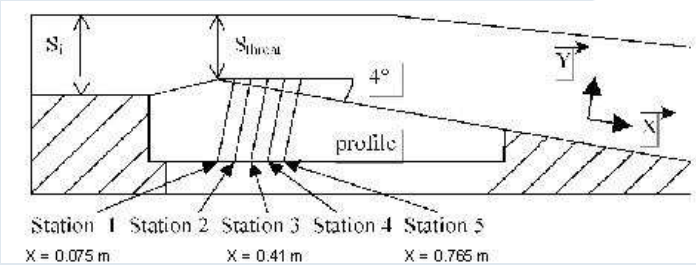
Experimental conditions

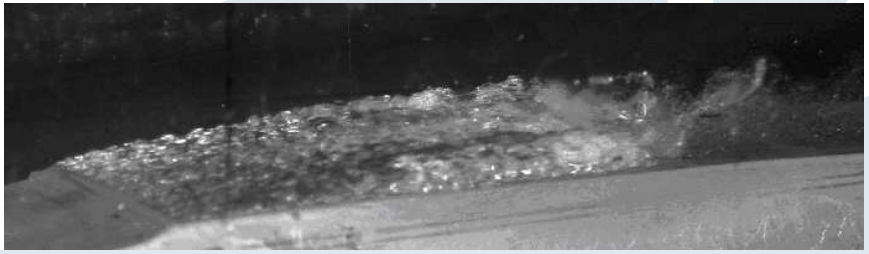
Operating point :

- $U_{inlet} = 10.8\ m/s$
- $\sigma_{inlet} = \frac{P_{inlet} - P_{vap}}{0.5\ \rho U_{inlet}^2} \approx 0.55$

Observations :

- A quasi stable cavitation of 0.70 to 0.85 m length
- An unsteady closure region with vapour cloud shedding and a liquid re-entrant jet



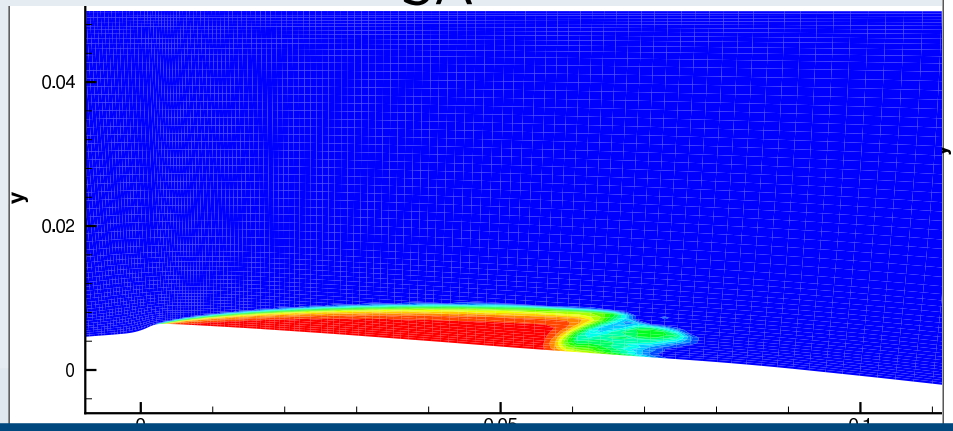


Measurements

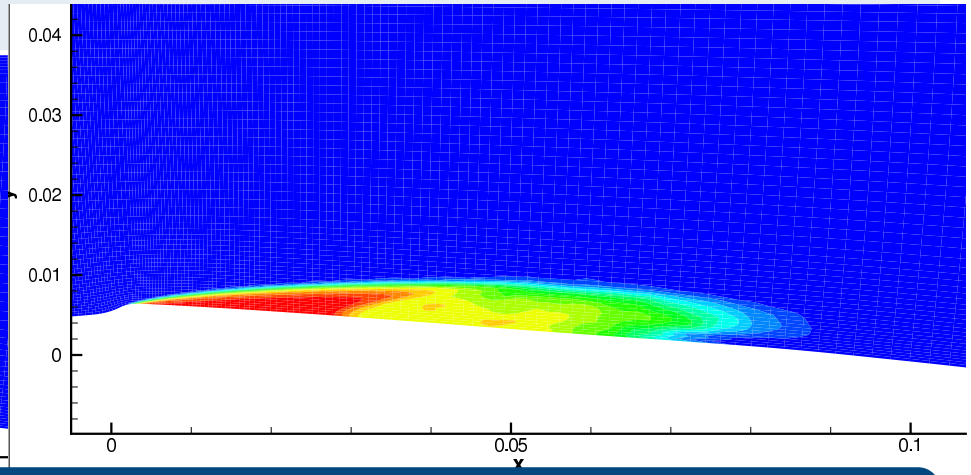
- Time-averaged longitudinal velocity and void ratio
- Time-averaged wall pressure evolution and RMS fluctuations

Results using the Spalart-Allmaras model, density gradient

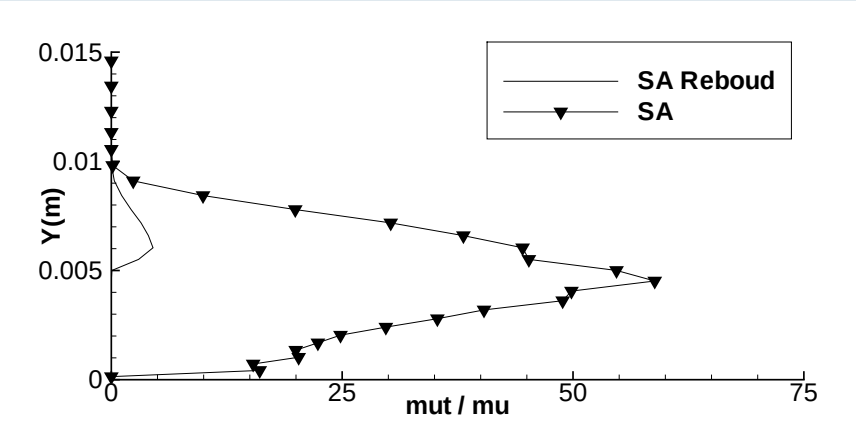
SA

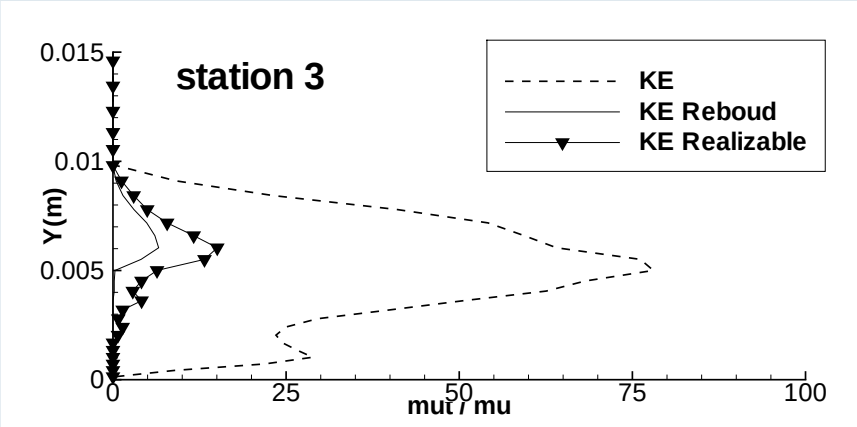


SA + Reboud limiter

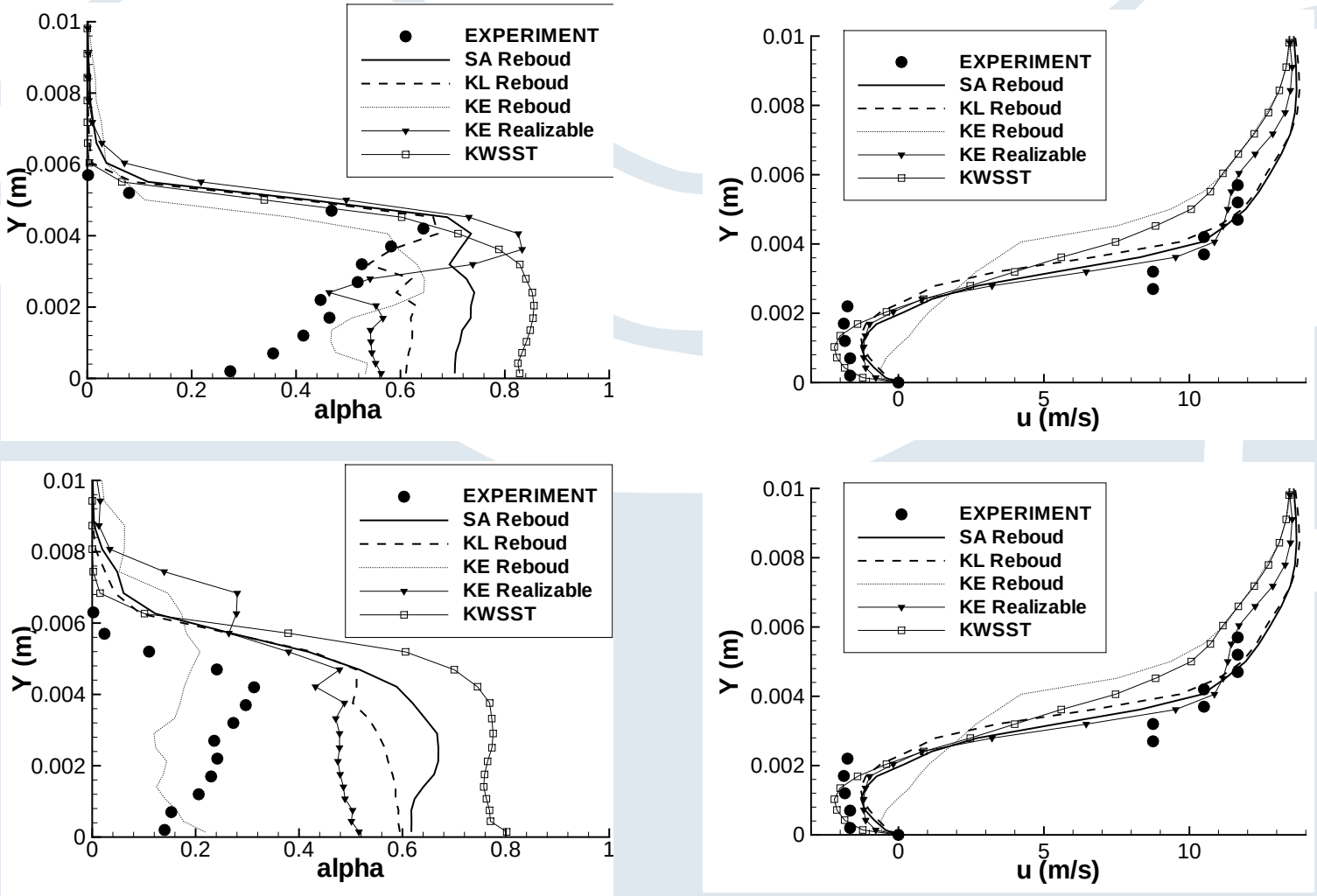


Limitation of the eddy viscosity μ_t



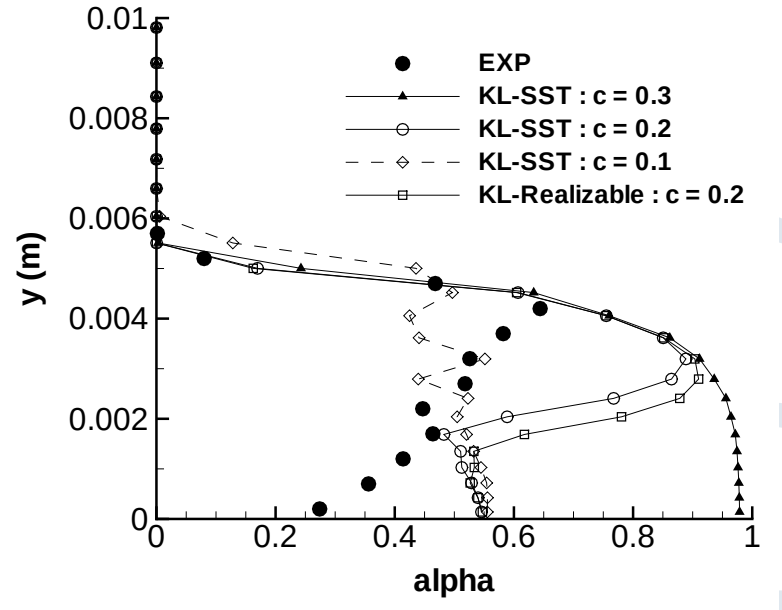


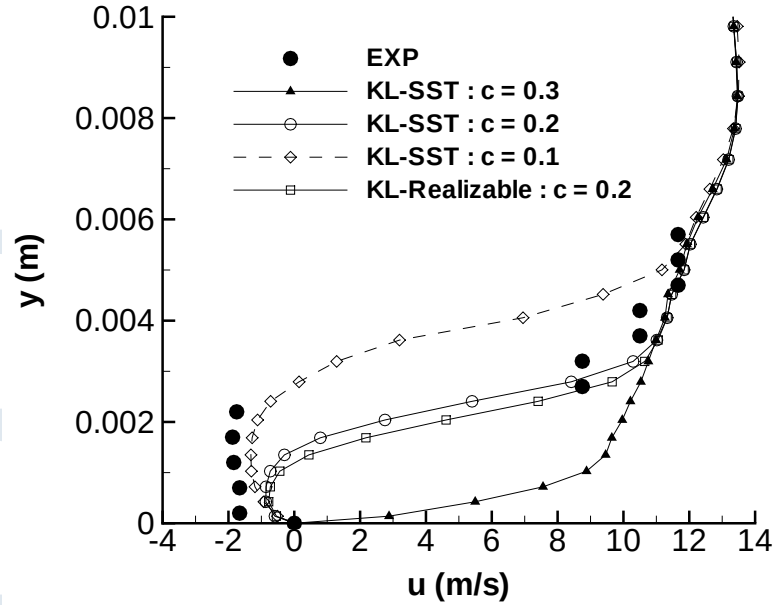
Eddy viscosity limiter, profiles at stations 3 and 4



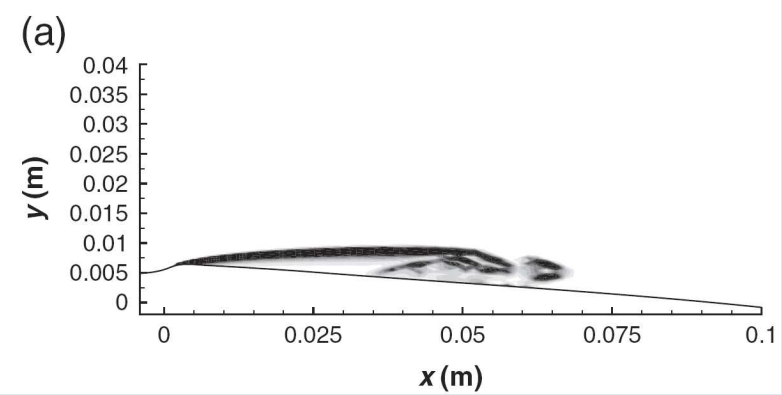
Time-averaged void fraction (left) and velocity (right) profiles

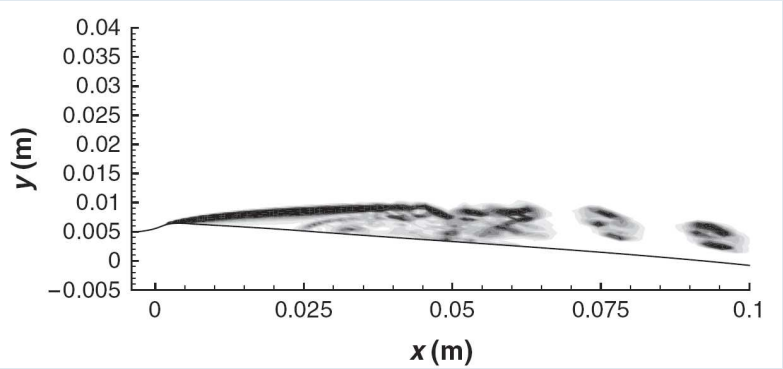
The $k - \ell$ model + SST correction





Time-averaged void fraction (left) and velocity (right) profiles





Contour of the density gradient : $a_1 = 0.2$ (left) and $a_1 = 0.1$ (right)

Compressibility corrections

Closure relations for turbulence compressible terms

Pressure-dilatation, Sarkar formulation

$$\rho \Pi = \overline{P' \frac{\partial u_i''}{\partial x_i}} = -\alpha_2 \rho P M_t + \alpha_3 \rho \varepsilon_s M_t^2$$

α_2, α_3 are constants to calibrate. $M_t = \frac{\sqrt{k}}{2c}$ is the turbulent Mach number.

Dilatational dissipation, Sarkar formulation

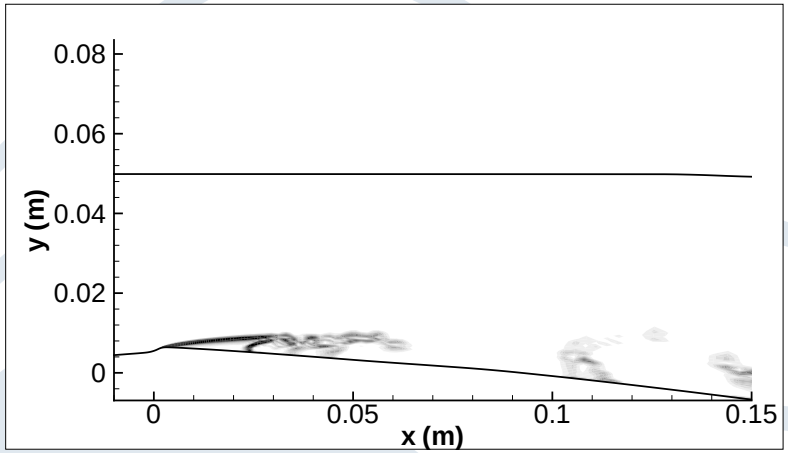
$$\varepsilon_d = \frac{4}{3} \mu \overline{s'_{kk} s'_{ll}} = \alpha_1 \varepsilon_s M_t^2 \quad \text{with } \alpha_1 \text{ to calibrate}$$

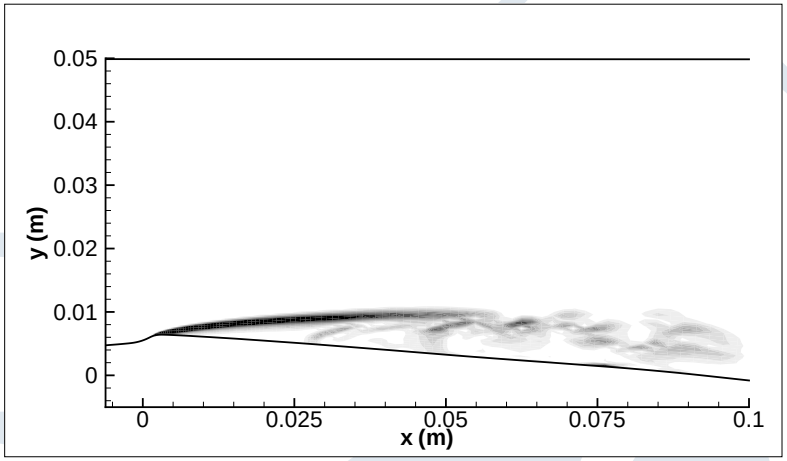
Mass flux, Jones formulation

$$\rho M = \frac{\overline{\rho' u_i''}}{\rho} \left(\frac{\partial \overline{P}}{\partial x_i} - \frac{\partial \overline{\sigma}_{il}}{\partial x_l} \right) = -\frac{\mu_t}{\rho^2 \sigma_p} \frac{\partial \rho}{\partial x_i} \frac{\partial P}{\partial x_i}$$

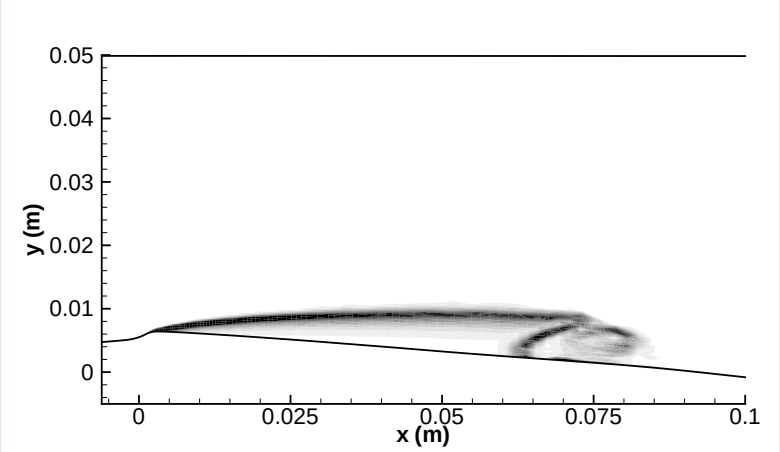
σ_p is a turbulent Schmidt number, which has to be calibrated.

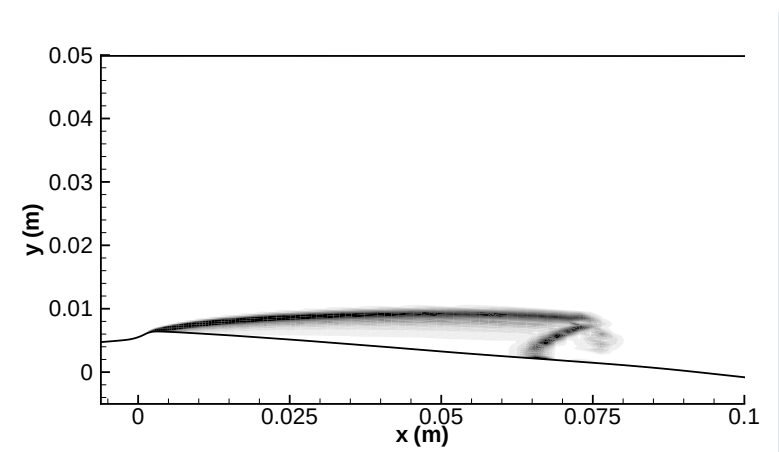
$k - \varepsilon$ compressible model





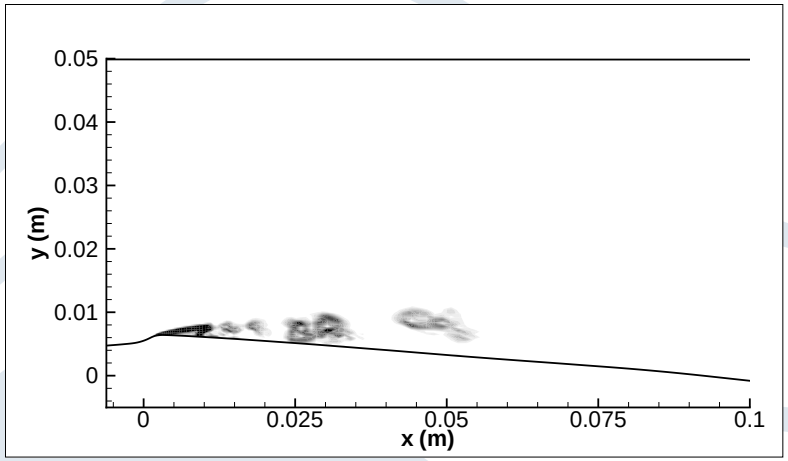
$k - \varepsilon + \Pi$, α_2=0.15, $\alpha_3 = 0.001$ (left) and $\alpha_3 = 0.025$ (right)

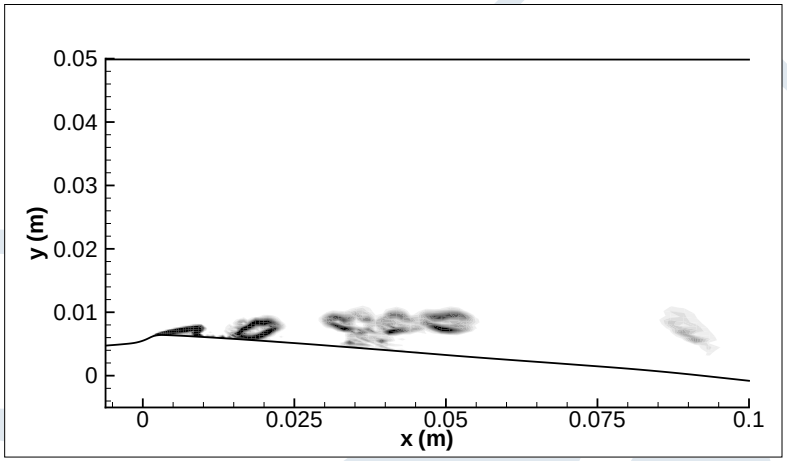




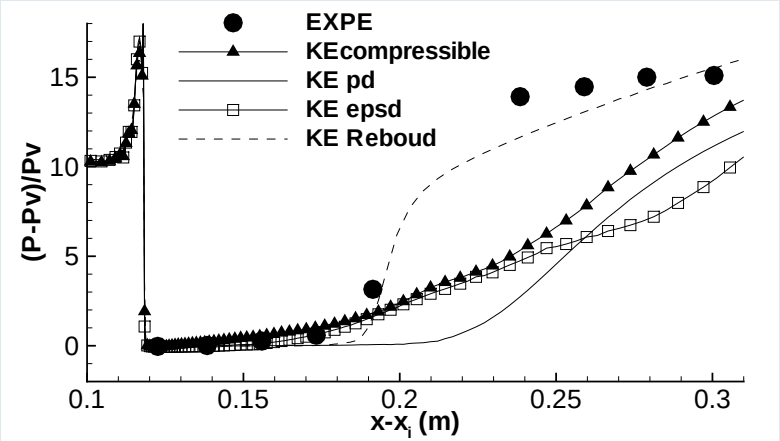
$k - \epsilon$ + mass flux term, $\sigma_p = 1$ (left) and $\sigma_p = 0.0001$ (right)

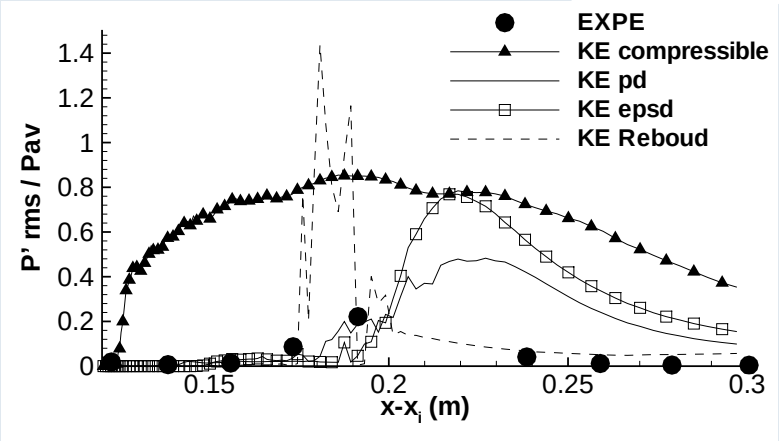
$k - \varepsilon$ compressible model (2)





$k - \epsilon + \Pi + M + \epsilon_d$, Sarkar values, at 2 instants





Time-averaged wall pressure (left) and RMS fluctuations (right)

Wall functions

Opened questions

- Existence of an universal velocity profile.
- Instantaneous logarithmic area.
- Cavitating law of the wall (κ function of α).
- Turbulence damping functions.
- Modifications of turbulent properties downstream a pocket.
- Compressibility effects.

Numerical study

- Computations using various meshes : y^+ from 1 to 50.
- Comparison of wall functions : two-layer model versus TBLE model.

Formulation - similar to single-phase flows

$$u^+ = y^+ \quad \text{if} \quad y^+ < 11.13$$

$$u^+ = \frac{1}{\kappa} \ln y^+ + 5.25 \quad \text{if} \quad y^+ > 11.13$$

where $\kappa = 0.41$ is the von Karman constant
For unsteady flows, validity of the velocity profile at each instant.

Turbulent quantities : the production of k or directly k is fixed following the formulation by Viegas and Rubesin :

$$P_k = \frac{1}{y} \int_0^y \tau_{xy}^t \frac{\partial u}{\partial y} \, dy$$

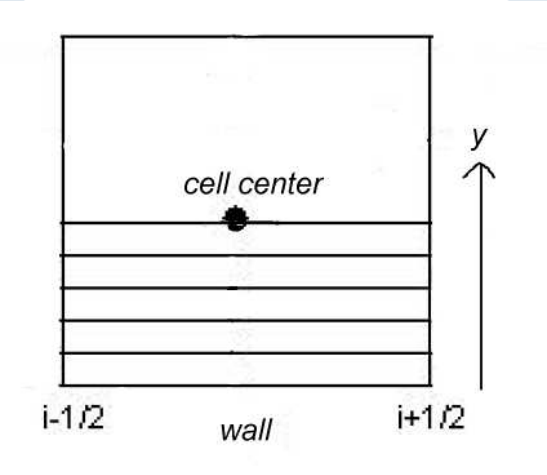
The second variable is computed through a length scale.

Formulation

Simplified momentum equation :

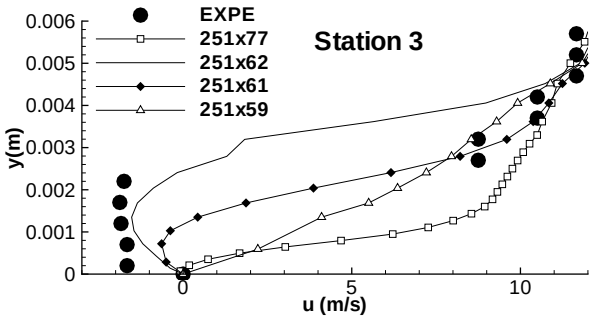
$$\frac{\partial u_i}{\partial t} + \frac{\partial u_i u_j}{\partial x_j} + \frac{1}{\rho}\frac{dP}{dx_i} = \frac{\partial}{\partial y}\left[(\mu + \mu_t)\frac{\partial u_i}{\partial y}\right]$$

Use of an embedded grid between the first grid point and the wall.

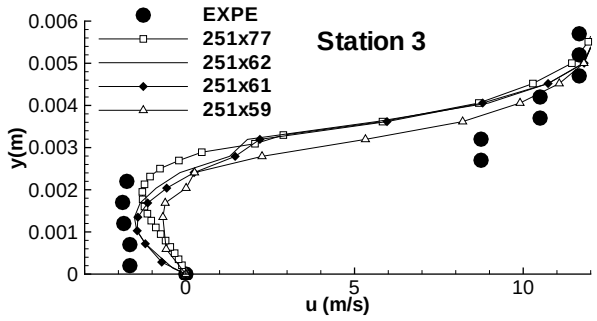


Discretization and integration of TBL equations in the embedded mesh. Iterative solving (Newton algorithm) for the variable τ_w. The number of nodes in the embedded grid is N =30.

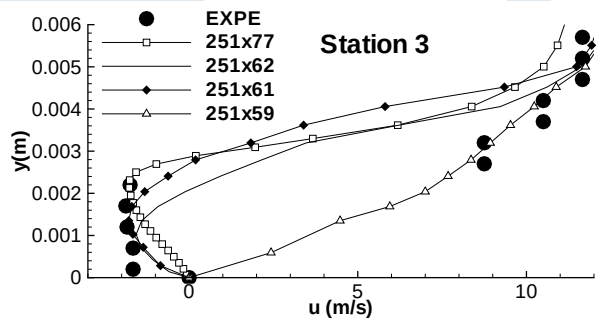
Venturi simulations



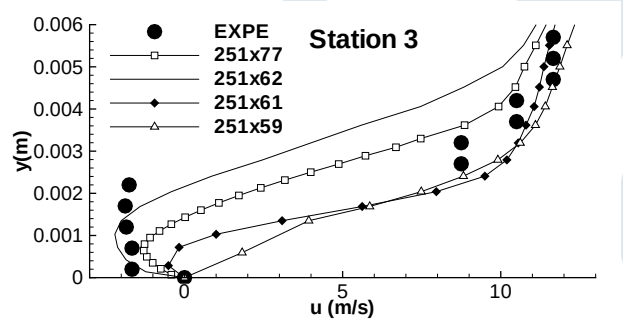
SA



KL

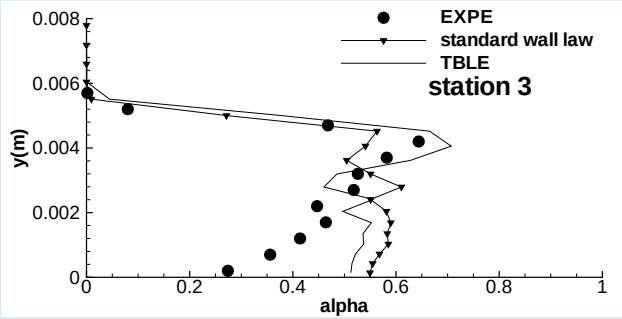


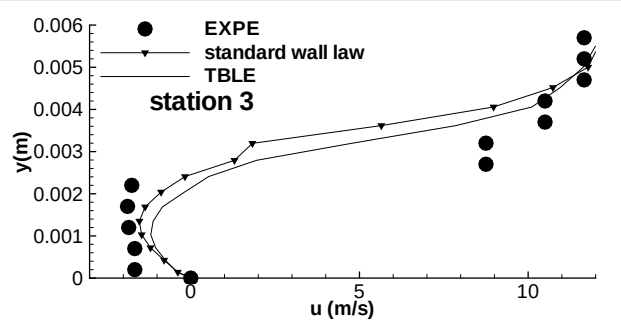
KE



KWSST

Mesh influence near wall, station 3





Two-layer model versus TBLE model

Frequency and CPU cost

The experimental frequency is around 45 Hz.
The inlet cavitation parameter $\sigma_{inlet} \sim 2.15$.

mesh	σ_{inlet}	frequency (Hz)	cost for 100 ite. (s)	ratio
174×77	2.13	30	525.6	1
	2.18	no frequency		
174×62	2.13	35	429.6	0.817
	2.17	40		
174×60	2.13	35	419.4	0.80
	2.19	43.5		
174×59	2.14	44	408	0.776
174×57	2.145	46	393	0.747
174×56	2.14	46	387	0.736

Improved model and hybrid RANS/LES

Framework

Starting point : the $k - k\ell$ model of Rotta (1972) where $k\ell = \frac{3}{16}\int_{-\infty}^{\infty} R_{ii}\left(\vec{x}, r_y\right) dr_y$

Correlation tensor $R_{ij}\left(\vec{x}, r_y\right) = \overline{U_i\left(\vec{x}\right)\ U_j\left(\vec{x} + r_y\right)}$
velocity correlation between a fixed point and a moving point in direction y

Transport equation for $\Psi = k\ell$ involving the integral quantity I :

$$I = -\frac{3}{16}\int_{-\infty}^{\infty} \frac{\partial U\left(\vec{x} + r_y\right)}{\partial y} R_{12} dr_y$$

Expansion in Taylor series gives :

$$I \approx \frac{\partial U\left(\vec{x}\right)}{\partial y}\int_{-\infty}^{\infty} R_{12} dr_y + \frac{\partial^2 U\left(\vec{x}\right)}{\partial y^2}\int_{-\infty}^{\infty} R_{12} r_y dr_y + \frac{1}{2}\frac{\partial^3 U\left(\vec{x}\right)}{\partial y^3}\int_{-\infty}^{\infty} R_{12} r_y^2 dr_y + \ldots$$

The second order derivative is neglected (assumption of homogeneous flow). The third order derivative is difficult to model and Rotta neglected also this term.

Menter modelling

Menter proposed a model using the second derivative of the velocity (indicator of the heterogeneity of the flow) and the von Karman scale L_{vk}

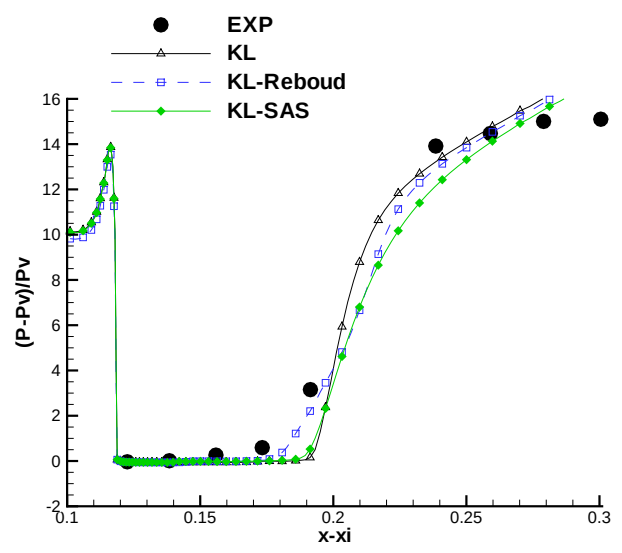
$$\frac{\partial^2 U}{\partial y^2}\int_{-\infty}^{\infty} R_{12} r_y dr_y \approx P_k \frac{k\ell}{k}\left(\frac{\ell}{L_{vk}}\right)^2 \qquad \text{and} \qquad L_{vk} = \kappa \left|\frac{U'}{U''}\right|$$

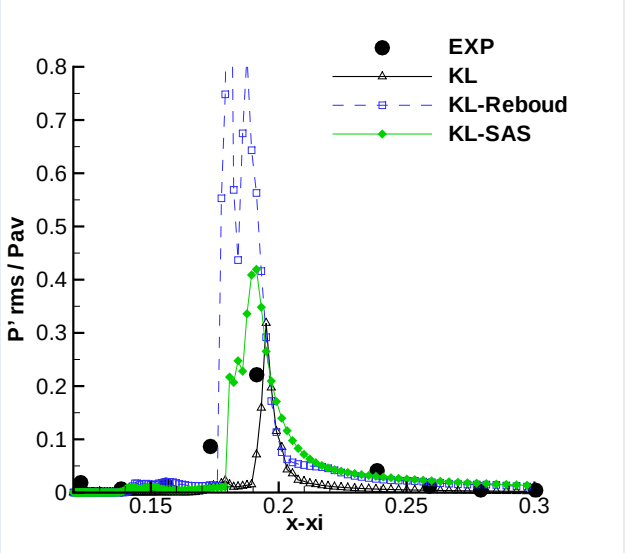
New term in the transport equation for Ψ (S-A term), driven by a constant ξ.
$\xi = 1.47$ following the calibration of Menter.

Scale-adaptive model

- The characteristic length scale is self-adaptive, function of the von Karman scale for a standard model, the length scale is proportional to δ
- It allows to adjust the solve of turbulent structures $\rightarrow$ behaviour close to a LES
- <u>BUT</u> : problem in the near-wall area, the log zone is not respected
  $\rightarrow$ the S-A term is not activated in the near-wall area
- calibration of ξ in cavitating flows.

Comparison between Reboud and S-A $k-\ell$ models

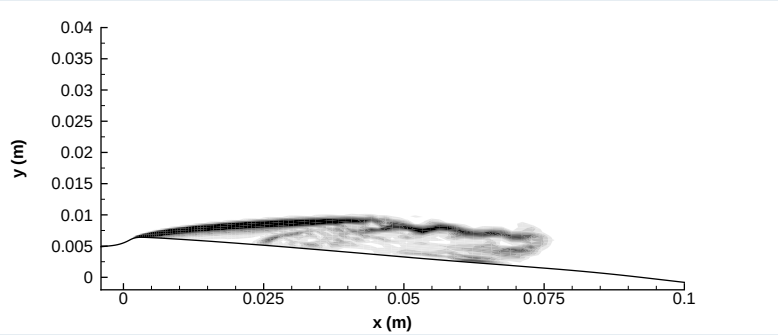




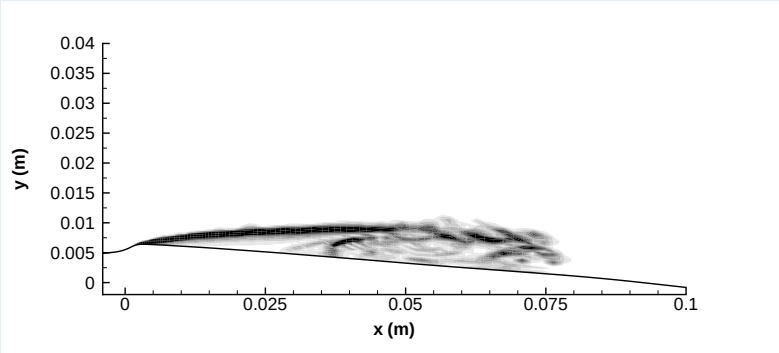
- time-averaged profiles quasi similar
- better pressure fluctuations with S-A model

Density gradient, Schlieren visualization

$k-\ell$ SAS



$k-\ell$ Reboud



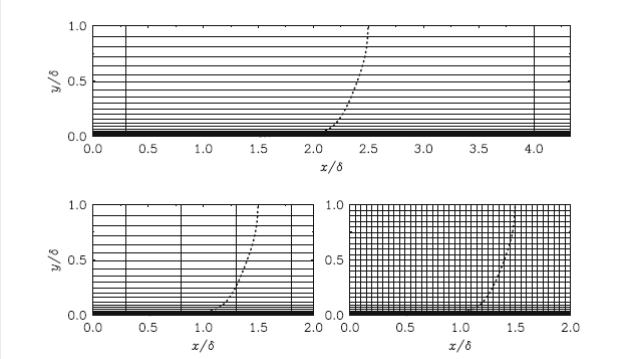
Detached Eddy Simulation of Spalart

$$\frac{\partial \rho\tilde{\nu}}{\partial t} + div\left[\vec{V}\rho\tilde{\nu} - \frac{1}{\sigma}\left(\mu + \rho\tilde{\nu}\right)\vec{grad}\,\tilde{\nu}\right] = c_{b1}\left(1 - f_{t2}\right)\tilde{S}\rho\tilde{\nu} + \frac{c_{b2}}{\sigma}\vec{grad}\,\rho\tilde{\nu}.\vec{grad}\,\tilde{\nu}$$
$$- \left(c_{\omega 1} f_{\omega} - \frac{c_{b1}}{\kappa^2} f_{t2}\right)\rho\frac{\tilde{\nu}^2}{\tilde{d}^2}$$

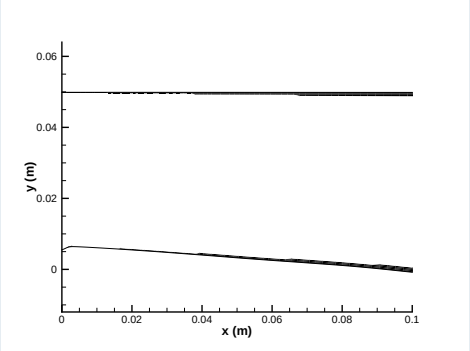
with $\tilde{d} = \min\left(d\ ,\ C_{DES}\Delta\right)$ and $\Delta = max(\Delta x, \Delta y, \Delta z)$.
C_{DES} is a constant evaluated for the decay of isotropic turbulence =0.65
In equilibrium area : $\tilde{\nu} = C_{DES}^2\Delta^2 S$

Drawbacks

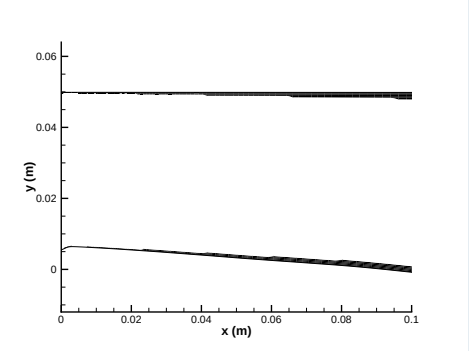
- Grid induced separation.
- Transition between RANS-mode and LES-mode : "grey" zone.
- Calibration of the constant C_{des}.



Influence of C_{DES}. RANS regions (black) and LES (white)

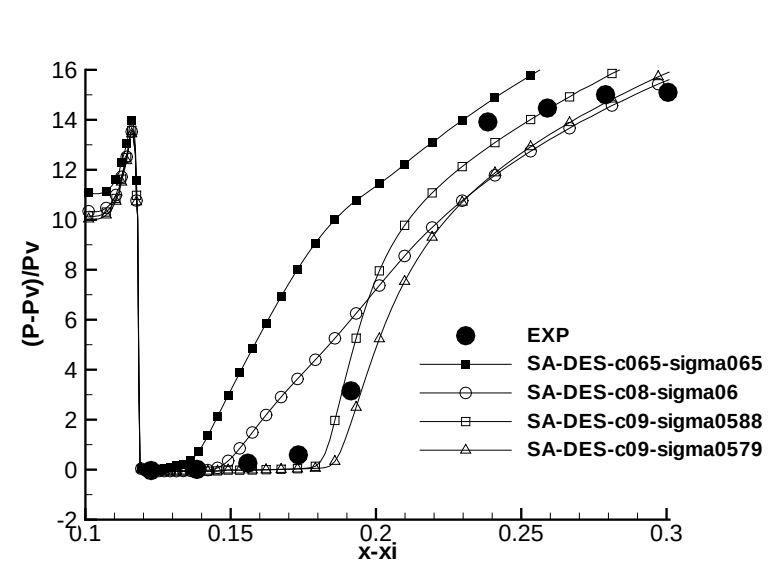


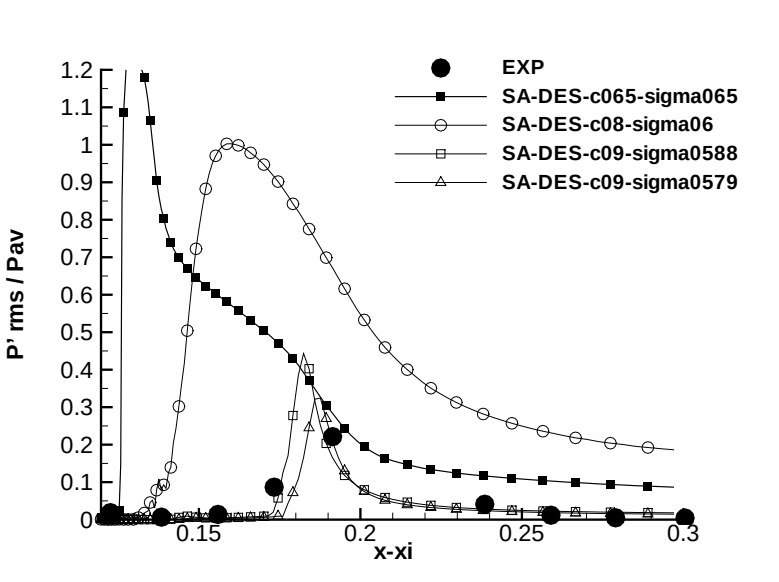
$Cdes = 0.65$



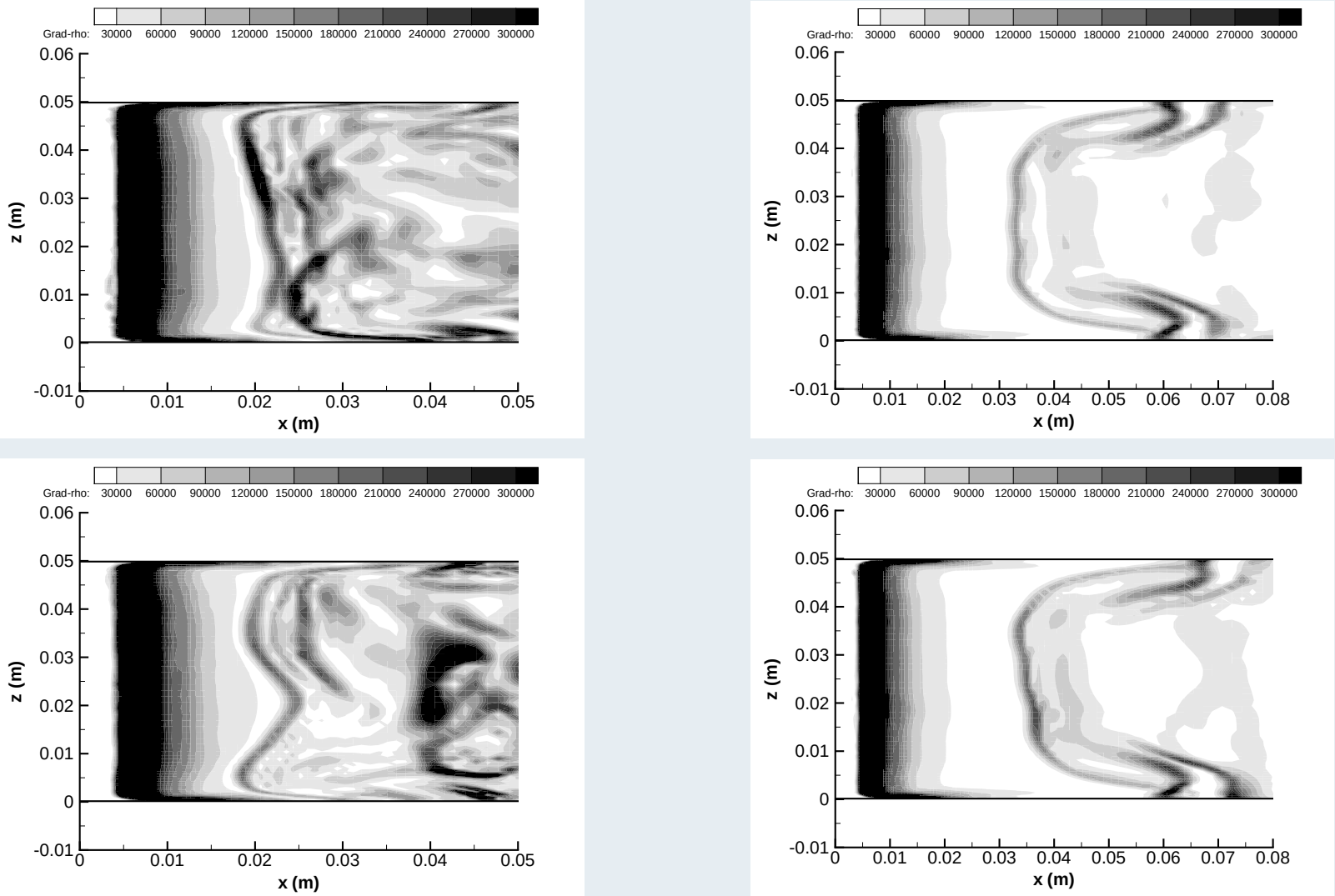
$Cdes = 0.9$

Wall pressure and RMS fluctuations



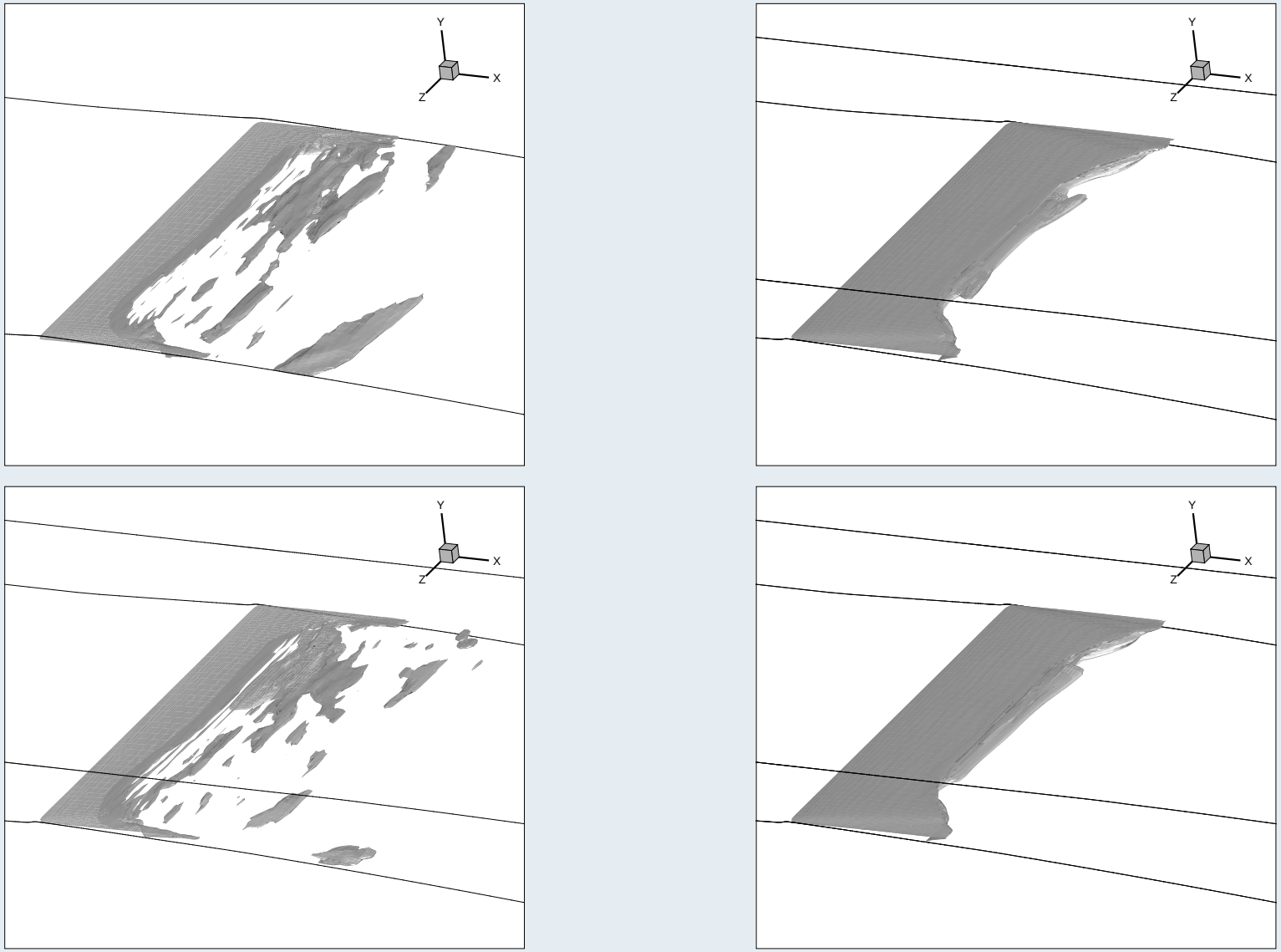


Comparison between $k-\ell$ S-A and DES simulations



gradient density visualization at two different instants : $k-\ell$ S-A (left) and DES (right)

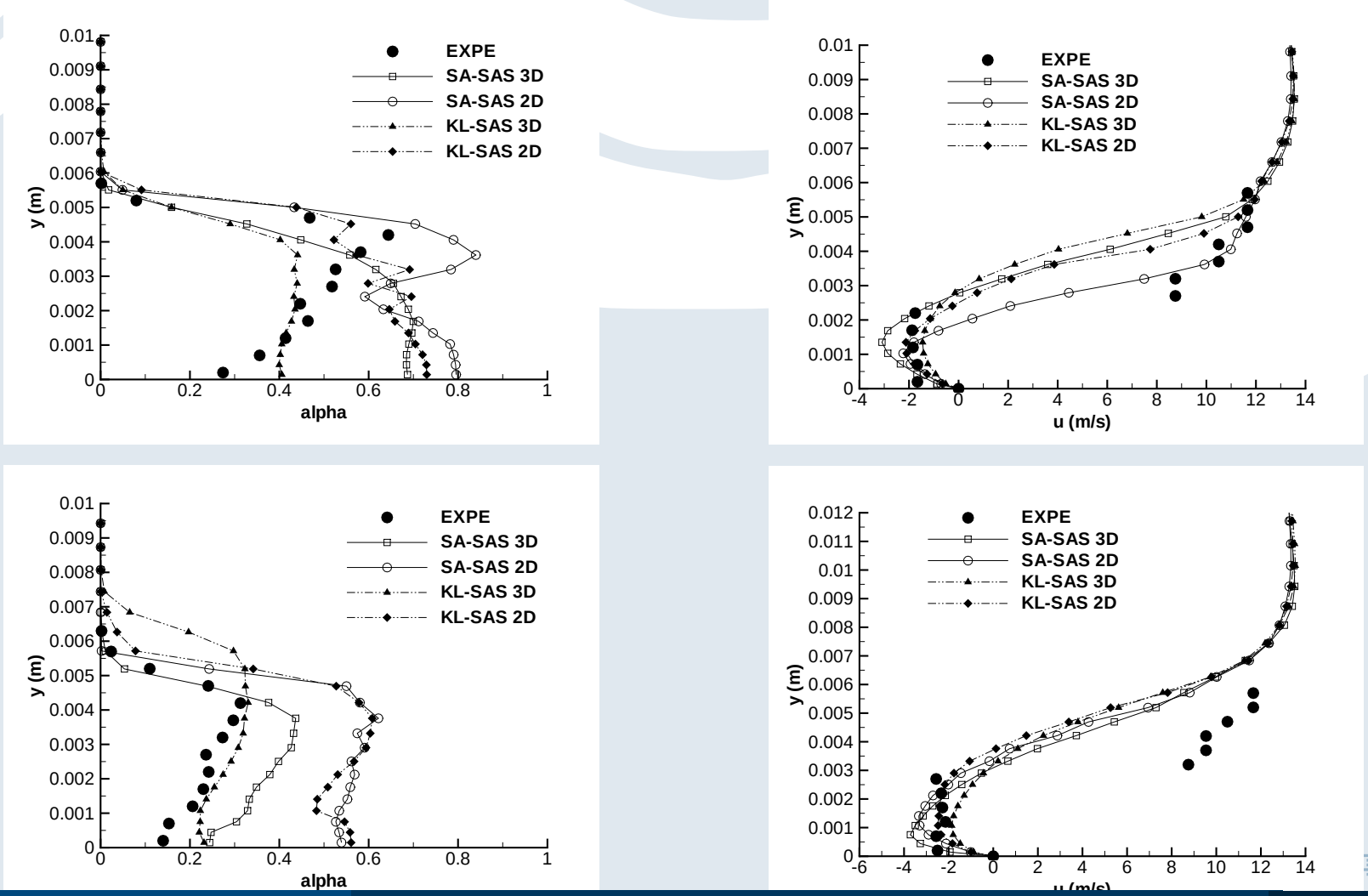
Comparison between $k - \ell$ Scale-Adaptive and DES simulations



Iso-surface of the void fraction for the value of 60% at two different

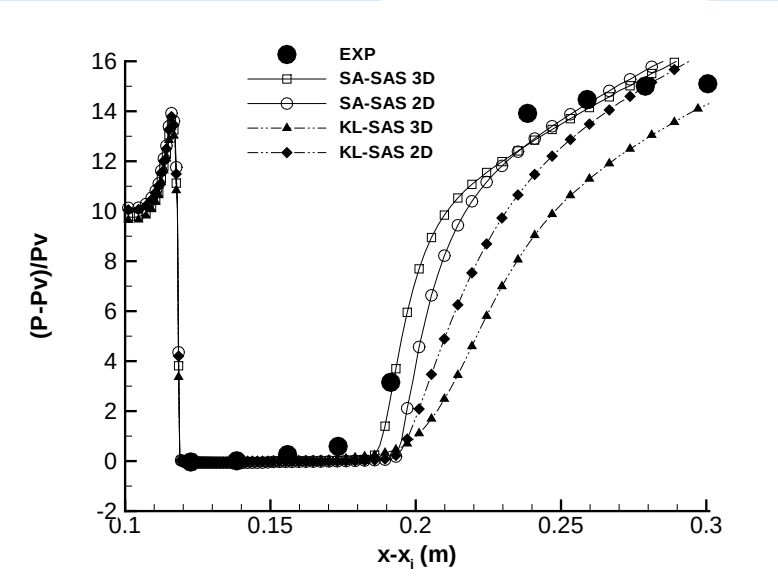
Comparison between 2D and 3D simulations

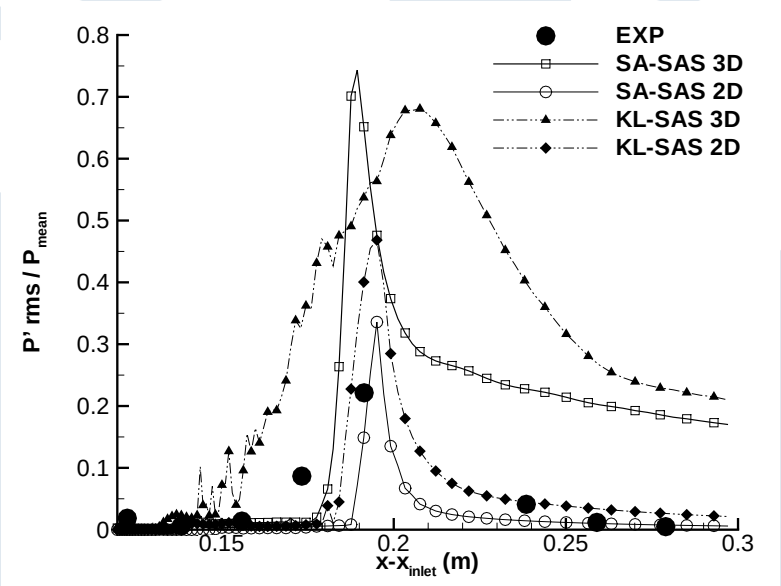
- Good agreement between models and experiment (mid-span profiles)
- 3D computations provides void fraction lower than 2D computations



Comparison between 2D and 3D simulations

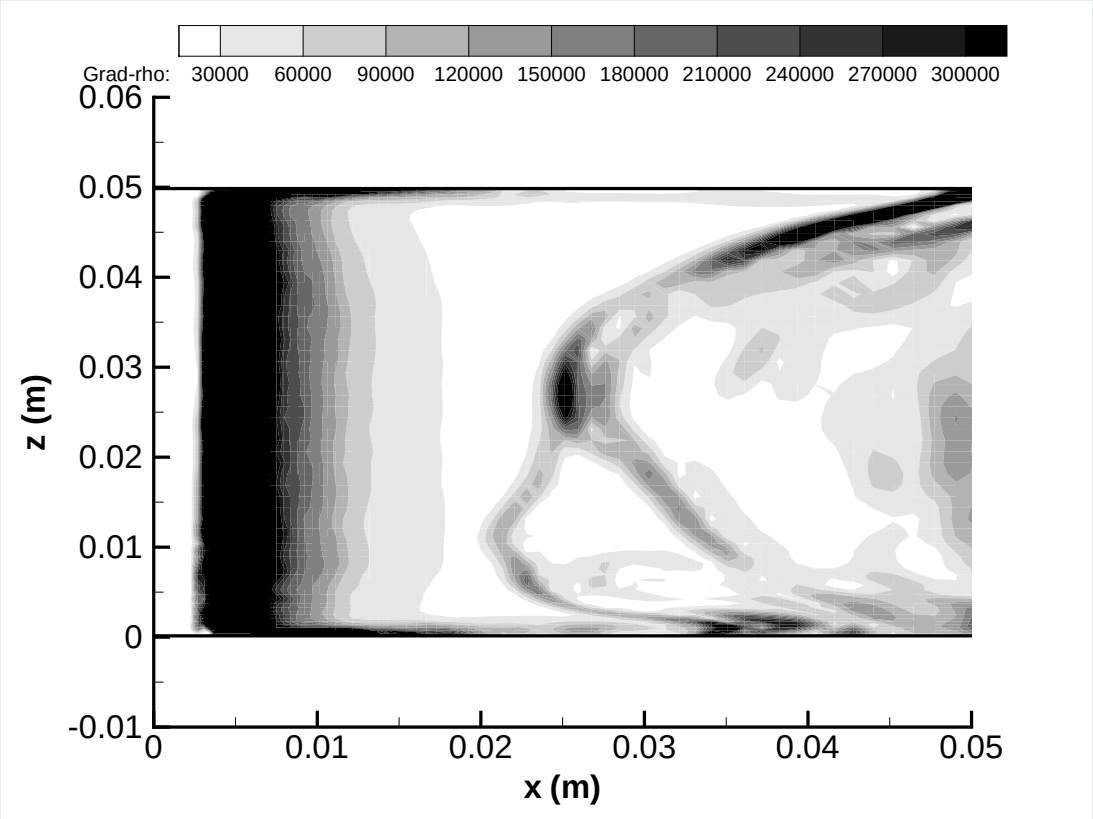
The level of RMS pressure fluctuations is largely overestimated by the 3D computations





Time-averaged wall pressure and RMS fluctuations

Dynamic behaviour - oblique mode using S-A simulations



Density gradient visualization

$\rightarrow$ a transversal instability at a low frequency : 6 Hz.

One-fluid filtered equations and LES

Scales

- Micro-scales, scales which are small enough to describe individual bubble shapes.
- Meso-scales, which are comparable to bubble sizes.
- Macro-scales, which entail enough bubbles for statistical representation.
- When LES is applied at a micro-scale, combination with interface tracking methods (see Lakehal).
- When LES is applied at a macro-scale, the interface resolution is not considered.
- The scale separation is mathematically obtained by applying a convolution product using a large-scale-pass filter (function G).

Filter

- For a quantity ϕ, the filtered variable is defined as : $\overline{\phi} = G \circ \phi$
- The Favre filtered variable : $\tilde{\phi} = \overline{\rho\phi}/\overline{\rho}$
- The filtered phase indicator function $\overline{X_k} = G \circ X_k = \alpha_k$ can be interpreted as a filtered volume fraction of phase k.

Phase indicator function

The phase indicator function is defined as :

$$X_k(M(x,t)) = \begin{cases} 1 & \text{if phase k is present in point } M(x,t) \text{ at } t \\ 0 & \text{otherwise} \end{cases}$$

Equations

Assumptions : the filtering operator commutes with time and spatial derivatives. The mass transfer is assuming to be proportional to the velocity divergence through a constant C.

$$
\begin{aligned}
\frac{\partial \overline{\rho}}{\partial t} + \frac{\partial \left(\overline{\rho}\tilde{u}_i\right)}{\partial x_j} &= 0 \\
\frac{\partial \overline{\rho}\tilde{u}_i}{\partial t} + \frac{\partial \left(\overline{\rho}\tilde{u}_i\tilde{u}_j + \overline{P}\delta_{ij}\right)}{\partial x_j} &= \operatorname{div}\left(2\overline{\mu}\tilde{S} + 2\tau_{\mu S} - 2\tau_{\rho S} - \tau_{\rho uu}\right) \\
\frac{\partial \overline{\rho}\tilde{E}}{\partial t} + \frac{\partial \left(\overline{\rho}\tilde{H}\tilde{u}_i + \overline{Q}_i^v\right)}{\partial x_j} &= \operatorname{div}\left(2\tau_{\mu Su} - 2\overline{\mu}\,\tilde{V}\tau_{\rho S} - 2\overline{\mu}\tilde{S}\tau_{\rho u}\right) \\
&+ \operatorname{div}\left(4\overline{\mu}\tau_{\rho u}\tau_{\rho S} + \overline{\rho}\tilde{H}\tau_{\rho u} - \tau_{\rho Hu}\right) \\
\frac{\partial \alpha}{\partial t} + \tilde{V}.\nabla\alpha - C\frac{\partial \tilde{u}_i}{\partial x_j} &= -\tau_{u\alpha} - C\operatorname{div}\left(\tau_{\rho u}\right)
\end{aligned}
$$

Subgrid terms

$$\begin{aligned}
\tau_{u\alpha} &= \overline{V.\nabla X_v} - \tilde{V}.\nabla\alpha \quad ; \quad \tau_{\rho u} = \tilde{V} - \overline{V} \\
\tau_{\mu S} &= \overline{\mu S} - \overline{\mu}\overline{S} \quad ; \quad \tau_{\rho S} = \tilde{S} - \overline{S} \\
\tau_{\rho Hu} &= \overline{\rho H V} - \overline{\rho}\tilde{H}\tilde{V} \quad ; \quad \tau_{\mu Su} = \overline{\mu S V} - \overline{\mu}\overline{S V} \\
\tau_{\rho uu} &= \overline{\rho}\left(\widetilde{V \otimes V} - \tilde{V} \otimes \tilde{V}\right)
\end{aligned}$$

- The subgrid term $\tau_{u\alpha}$ is specific to two-phase flows.
- The influence and the hierarchy of all these terms have never been investigated in cavitating flows.
- The magnitude of the different subgrid terms was *a priori* evaluated in the case of phase separation flows and turbulence bubble interaction (Labourasse,Vincent) $\rightarrow$ the influence of $\tau_{u\alpha}$ is highly dependent on the flow configurations and/or on the chosen two-phase description.

Modelling difficulties

- Lots of assumption in models.
- Lack of experimental data or DNS.
- Compressibility turbulent closure due to the no divergence free fluctuating velocity field
- Advanced turbulence models and hybrid turbulence model to improve the level of resolved scales
- A challenge : LES in cavitating flows.