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Eric Goncalvès da Silva

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Eric Goncalvès da Silva. Turbulence modelling in cavitating flows. Doctoral. Udine, France. 2014. cel-01420694

**HAL Id: cel-01420694**

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Submitted on 20 Dec 2016

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# Turbulence modelling in cavitating flows

Eric Goncalves

LEGI, University of Grenoble-Alpes, France

7-11 July 2014



LABORATOIRE DES ECOULEMENTS  
GÉOPHYSIQUES ET INDUSTRIELS

## Outline

- 1 General considerations
- 2 One-fluid RANS equations
- 3 Limitation of eddy viscosity
- 4 Compressibility terms
- 5 Wall models
- 6 Improved models - hybrid RANS/LES
- 7 One-fluid filtered equations and LES
- 8 Conclusions

# General considerations

## Questions

- Kolmogorov spectrum, slope  $\neq -5/3$
- Kolmogorov scale / size of two-phase structures.
- Induced turbulence or pseudo-turbulence.
- Compressibility effects on turbulence.
- Anisotropy of the Reynolds tensor.
- Increase or decrease of the turbulence intensity.
- Cavitation-turbulence interaction at small scales.

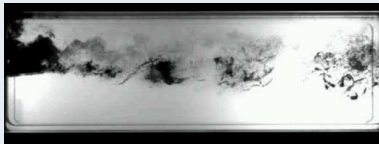
## Remarks

- no DNS data or turbulent quantities measurements.
- non universality of two-phase flows.

## Cavitation pockets



## Cavitation in vortex



## Turbulence scales

- DNS : all scales are resolved
- LES : large scale are resolved and small scales are modelled
- RANS : mean flow is resolved and turbulence is modelled
- Hybrid RANS/LES : model adapts to the mesh resolution

## Phase scales

- DNS : resolution of each fluid + interface
- pseudo-DNS : resolution of each fluid + tracking of the interface (VoF, level set...)
- filtered (LES) and averaged (RANS) models :
  - two-fluid model,
  - reduced models,
  - one-fluid model

## Averaged one-fluid equations (RANS)

- Temporal averaged or ensemble averaged equations
- The same operator for two-phase structure and turbulent structure
- Boussinesq analogy used similarly to single-phase flows
- Transport-equation models :  $k - \varepsilon$ ,  $k - \omega$ ,...

## Filtered one-fluid equations (LES)

- First works in incompressible flow without phase transition
- Lots of sub-grid terms - problem of modelling
- Link with the DNS





# Averaged equations

## Temporal phase average

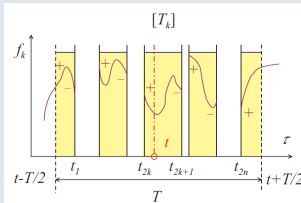
Averaging over a time  $T$  :

$$\overline{\phi}_k = \frac{1}{T} \int_T \phi_k(x, \tau) d\tau$$

Averaging over the time  $T_k$  of presence of the phase  $k$  :

$$\overline{\phi}_k = \frac{1}{T_k} \int_{T_k} \phi_k(x, \tau) d\tau$$

$$\text{Void fraction : } \alpha(x, t) = \frac{T_k}{T} = \frac{\overline{\phi}_k}{\overline{\phi_k}}$$



## Mass-weighted average (Favre average)

For a variable  $\rho\phi$ , a mass-weighted average :

$$\tilde{\phi}_k = \frac{\overline{\rho_k \phi_k}}{\overline{\rho_k}} = \frac{\overline{\overline{\rho_k \phi_k}}}{\overline{\overline{\rho_k}}} \quad \text{and} \quad \tilde{\phi} = \frac{\sum \alpha_k \overline{\overline{\rho_k \phi_k}}}{\sum \alpha_k \overline{\overline{\rho_k}}} = \frac{\sum \overline{\overline{\rho_k \phi_k}}}{\sum \overline{\overline{\rho_k}}}$$

## Mass-weighted decomposition

An average part and a fluctuation (two-phase and turbulent contribution) :

$$\begin{aligned} \rho_k &= \overline{\overline{\rho_k}} + \rho'_k, & u_k &= \tilde{u}_k + u''_k, & \phi_k &= \tilde{\phi}_k + \phi''_k \\ \overline{\overline{\rho'_k}} &= 0, & \overline{\overline{\rho_k u''_k}} &= 0, & \overline{\overline{\rho_k \phi''_k}} &= 0 \end{aligned}$$

## Conservative equations for the phase $k$

Mass conservation :

$$\frac{\partial \alpha_k \bar{\rho}_k}{\partial t} + \text{div} (\alpha_k \bar{\rho}_k \tilde{u}_k) = \Gamma_k$$

Momentum conservation :

$$\frac{\partial \alpha_k \overline{\rho_k \tilde{u}_k}}{\partial t} + \text{div} (\alpha_k \bar{\rho}_k \tilde{u}_k \otimes \tilde{u}_k + P_k) = \text{div} (\alpha_k (\bar{\tau}_k + \tau_k^t)) + \mathbb{M}_k$$

Total energy conservation :

$$\begin{aligned} \frac{\partial \alpha_k \bar{\rho}_k (\tilde{E}_k + k_k)}{\partial t} + \text{div} (\alpha_k \rho_k (\tilde{E}_k + k_k) \tilde{u}_k) &= \text{div} (\alpha_k P_k \tilde{u}_k) + \text{div} (\alpha_k (\bar{\tau}_k + \tau_k^t) \tilde{u}_k) \\ &\quad - \text{div} (\alpha_k (\bar{q}_k + q_k^t)) + \mathbb{Q}_k \end{aligned}$$

New unknowns : Reynolds stress tensor  $\tau_k^t$  and turbulent heat flux  $q_k^t$

## Fluctuating fields

The velocity fluctuating field is not divergence free (even with incompressible phases) :

$$\frac{\partial u'_{k,l}}{\partial x_l} = -\frac{\partial \bar{u}_{k,l}}{\partial x_l} = \frac{1}{\alpha_k} u'_k \cdot n_k \delta_l$$

It leads to supplementary unknowns, called "compressible terms".  
Equation for the momentum fluctuation :

$$\begin{aligned} \rho_k \frac{\partial u''_{k,i}}{\partial t} + \rho_k \tilde{u}_{k,l} \frac{\partial u''_{k,i}}{\partial x_l} &+ \rho_k u''_{k,l} \frac{\partial \tilde{u}_{k,i}}{\partial x_l} + \rho_k u_{k,l} \frac{\partial u''_{k,i}}{\partial t} \\ &= \frac{\partial \mathbb{T}_{k,il}}{\partial x_l} + \rho_k \tilde{u}_{k,i} \frac{\partial \tilde{u}_{k,l}}{\partial x_l} \\ &+ \frac{\rho_k}{\bar{\rho}_k \alpha_k} \left[ -\frac{\partial \alpha_k \bar{\mathbb{T}}_{k,il}}{\partial x_l} - \mathbb{M}_k + \frac{\partial}{\partial x_l} \left( \alpha_k \bar{\rho}_k \widetilde{u''_{k,i} u''_{k,l}} \right) \right] \\ &+ \frac{\rho_k}{\bar{\rho}_k \alpha_k} \left[ \tilde{u}_{k,i} \left( \frac{\partial \alpha_k \bar{\rho}_k}{\partial t} + \tilde{u}_{k,l} \frac{\partial \alpha_k \bar{\rho}_k}{\partial x_l} \right) \right] \end{aligned}$$

# Turbulent kinetic energy equation for the phase $k$

it is possible to write the equation for the phasic Reynolds stress and the phasic TKE  $k_k$

$$\frac{\partial \alpha_k \bar{\rho}_k k_k}{\partial t} + \text{div} (\alpha_k \bar{\rho}_k k_k \tilde{u}_k) = \alpha_k \bar{\rho}_k (P_k - \epsilon_k + \Pi_k + M_k + D_k) + \Gamma_k K^\Gamma$$

with :

$$\bar{\rho}_k \alpha_k P_k = -\alpha_k \bar{\rho}_k \widetilde{u''_{k,i} u''_{k,l}} \frac{\partial \tilde{u}_{k,i}}{\partial x_l} \quad \text{Production term}$$

$$\bar{\rho}_k \alpha_k \epsilon_k = \overline{\alpha_k \tau'_{k,il} \frac{\partial u''_{k,i}}{\partial x_l}} \quad \text{Dissipation rate}$$

$$\bar{\rho}_k \alpha_k \Pi_k = \overline{\alpha_k p'_k \frac{\partial u''_{k,i}}{\partial x_i}} \quad \text{Pressure-dilatation term}$$

$$\bar{\rho}_k \alpha_k M_k = \overline{\alpha_k u''_{k,i} \left[ -\frac{\partial \bar{p}_k}{\partial x_i} + \frac{\partial \bar{\sigma}_{k,il}}{\partial x_l} \right]} \quad \text{Mass flux term}$$

$$\bar{\rho}_k \alpha_k D_k = -\frac{\partial}{\partial x_l} \left[ \overline{\alpha_k P'_k u''_{k,i}} \delta_{il} - \overline{\alpha_k \tau'_{k,il} u''_{k,i}} \right] - \frac{\overline{\alpha_k \rho_k \frac{u''_{k,i} u''_{k,i}}{2} u''_{k,l}}}{\partial x_l} \quad \text{Diffusion term}$$

$$\Gamma_k K^\Gamma \quad \text{Mass transfer term}$$

## Mixture mean quantities

- mixture density and pressure :  $\rho_m = \sum \alpha_k \bar{\rho}_k$ ,  $P_m = \sum \alpha_k \bar{P}_k$
- mixture internal energy :  $\rho_m e_m = \sum \alpha_k \bar{\rho}_k \tilde{e}_k$
- mass center velocity :  $\rho_m u_{m,i} = \sum \alpha_k \bar{\rho}_k \tilde{u}_{k,i}$
- mixture viscosity :  $\mu_m = \sum \alpha_k \bar{\mu}_k$
- mixture viscous stress tensor :  $\tau_{m,ij} = \sum \alpha_k \overline{\tau_{k,ij}}$
- mixture heat flux :  $q_m = \sum \alpha_k \bar{q}_k$

## Mixture turbulent quantities

- mixture turbulent kinetic energy :  $k_m = \sum \alpha_k k_k = \sum \alpha_k \widetilde{u_{k,i}''^2} / 2$
- mixture Reynolds stress tensor :  $\tau_{m,ij}^t = - \sum \alpha_k \overline{\rho_k u_{k,i}'' u_{k,j}''}$
- mixture eddy viscosity :  $\mu_{tm} = \sum \alpha_k \mu_{tk}$
- mixture turbulent heat flux :  $q_m^t = \sum \alpha_k q_k^t$
- mixture dissipation rate :  $\varepsilon_m$ , mixture specific dissipation  $\omega_m \dots$

## Conservative equations

Mass equation :

$$\frac{\partial \rho_m}{\partial t} + \operatorname{div}(\rho_m u_m) = 0$$

Momentum equation :

$$\frac{\partial \rho_m u_m}{\partial t} + \operatorname{div}(\rho_m u_m \otimes u_m + P_m) = \operatorname{div}(\tau_m + \tau_m^t)$$

Energy equation :

$$\begin{aligned} \frac{\partial \rho_m (E_m + k_m)}{\partial t} + \operatorname{div}(\rho_m (E_m + k_m) u_m) &= \operatorname{div}(-P_m u_m) - \operatorname{div}(q_m - q_m^t) \\ &+ \operatorname{div}[(\tau_m + \tau_m^t) u_m] \end{aligned}$$



## Mixture turbulent kinetic energy equation

$$\frac{\partial \rho_m k_m}{\partial t} + \text{div}(\rho_m k_m u_m) = \rho_m P_m + \rho_m \Pi_m - \rho_m \epsilon_m + \rho_m M_m + \rho_m D_m + \Gamma_m K_m^\Gamma$$

It is assumed that each phase shares the same fluctuating velocity  $u_i''$ , the same fluctuating pressure  $P'$  and the same fluctuating viscous stress  $\tau'_{ij}$  :

$$\begin{aligned} \rho_m P_m &= \tau_{m,il}^t \frac{\partial u_{m,i}}{\partial x_l} & ; & & \rho_m \Pi_m &= \overline{P' \frac{\partial u_i''}{\partial x_i}} \\ \rho_m \epsilon_m &= \overline{\tau'_{il} \frac{\partial u_i''}{\partial x_l}} & ; & & \Gamma_m K_m^\Gamma &= 0 \end{aligned}$$

$$\rho_m M_m = \overline{u_i''} \left[ -\frac{\partial \rho_m}{\partial x_i} \right] + \overline{u_i''} \left[ \frac{\partial \sigma_{m,il}}{\partial x_l} - \sum_k \overline{\overline{\sigma}}_{k,il} \frac{\partial \alpha_k}{\partial x_l} \right]$$

$$\rho_m D_m = -\frac{\partial}{\partial x_l} \left[ \overline{\rho' u_i''} \delta_{il} - \overline{\sigma'_{il} u_i''} \right] - \frac{\partial \rho_m \overline{\frac{u_i'' u_i''}{2} u_l''}}{\partial x_l}$$

## Main features

- A fluctuation is associated to a mixture quantity !
- The fluctuating velocity field is not divergence free  $\rightarrow$  supplementary terms, difficult to model.
- The pressure-dilatation term  $\rho_m \Pi_m$  is null in mean but not instantaneously.
- The mixture dissipation  $\varepsilon$  is not only solenoidal.  
Inhomogeneous and dilatational (or compressible) contributions.

$$\rho \varepsilon \approx \underbrace{2\mu \overline{\omega'_{ik} \omega'_{ik}}}_{\rho \varepsilon_s} + \underbrace{2\mu \frac{\partial}{\partial x_k} \left[ \frac{\partial \overline{u'_k u'_l}}{\partial x_l} - 2 \overline{u'_k s'_{ll}} \right]}_{\rho \varepsilon_{inh}} + \underbrace{\frac{4}{3} \mu \overline{s'_{kk} s'_{ll}}}_{\rho \varepsilon_d}$$

- The diffusion term is modelled with a gradient formulation.
- The mixture dissipation equation  $\varepsilon$  is completely modelled following the single-phase formulation.

### Boussinesq assumption

Boussinesq analogy and mixture eddy viscosity assumption  $\mu_{tm}$  :

$$\tau_{m,ij}^t = \mu_{tm} \left[ \frac{\partial u_{m,i}}{\partial x_j} + \frac{\partial u_{m,j}}{\partial x_i} - \frac{2}{3} \text{div } u_m \delta_{ij} \right] - \frac{2}{3} \rho_m k_m \delta_{ij}$$

Evaluation of the mixture eddy viscosity with transport-equation models.

### Turbulent Fourier law

Fourier law analogy and mixture thermal conductivity  $\lambda_m^t$  :

$$q_m^t = -\lambda_m^t \text{grad } T_m$$

Assumption of constant turbulent Prandtl number  $Pr_t$  :

$$\lambda_m^t = \sum \alpha_k \lambda_k^t = \sum \alpha_k \frac{\mu_k^t C_{pk}}{Pr_t} \quad \text{approximated by} \quad \lambda_m^t \simeq \frac{\mu_m^t C_{pm}}{Pr_t}$$

## Usual model, $k - \varepsilon$ model for a mixture

transport-equation models equivalent to single-phase turbulent models.  
all supplementary terms are neglected.  
only the solenoidal dissipation is taken into account.

$$\begin{aligned}\frac{\partial \rho k}{\partial t} + \operatorname{div} \left[ \rho u k - \left( \mu + \frac{\mu_t}{\sigma_k} \right) \operatorname{grad} k \right] &= \rho P_k - \rho \varepsilon \\ \frac{\partial \rho \varepsilon}{\partial t} + \operatorname{div} \left[ \rho u \varepsilon - \left( \mu + \frac{\mu_t}{\sigma_\varepsilon} \right) \operatorname{grad} \varepsilon \right] &= c_{\varepsilon 1} \frac{\varepsilon}{k} \rho P_k - \rho c_{\varepsilon 2} f_2 \frac{\varepsilon^2}{k} \\ \mu_t &\approx \rho \frac{k^2}{\varepsilon_s}\end{aligned}$$

## Remarks

- a large quantity of assumptions.
- introduction of wall treatment (damping functions or wall functions)
- a large quantity of problems !

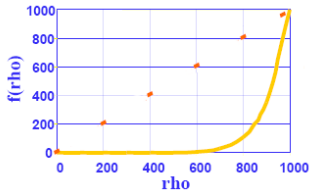
# Limitation of the eddy viscosity

## The Rebound correction

$$\mu_t = f(\rho) C_\mu \frac{k^2}{\epsilon}$$

$$f(\rho) = \rho_v + \left( \frac{\rho_v - \rho}{\rho_v - \rho_l} \right)^n (\rho_l - \rho_v)$$

$n$  is usually set to 10



## SST correction - Bradshaw's assumption for 2D boundary layer

$$\mu_t = \frac{\rho k / \omega}{\max\left(1, \frac{\Omega F_2}{a_1 \omega}\right)} ; a_1 = 0.3 ; \Omega = \sqrt{2\Omega_{ij}\Omega_{ij}} \text{ with } \Omega_{ij} = \frac{1}{2} \left( \frac{\partial \tilde{u}_i}{\partial x_j} - \frac{\partial \tilde{u}_j}{\partial x_i} \right)$$

## Realisability constrains of Durbin

$$\mu_t = \min\left(C_\mu^0 ; \frac{c}{s\sqrt{3}}\right) \frac{\rho k^2}{\epsilon} ; 0 \leq c \leq 1 ; C_\mu^0 = 0.09 ; s = \frac{k}{\epsilon} S \text{ with } S = 2S_{ij}S_{ij} - \frac{2}{3}S_{kk}^2$$

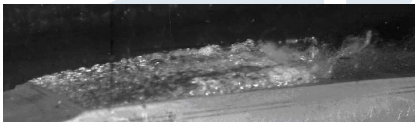
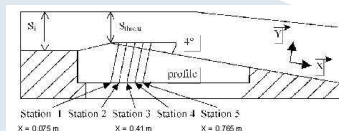
## Experimental conditions

Operating point :

- $U_{inlet} = 10.8 \text{ m/s}$
- $\sigma_{inlet} = \frac{P_{inlet} - P_{vap}}{0.5 \rho U_{inlet}^2} \approx 0.55$

Observations :

- A quasi stable cavitation of 0.70 to 0.85 m length
- An unsteady closure region with vapour cloud shedding and a liquid re-entrant jet

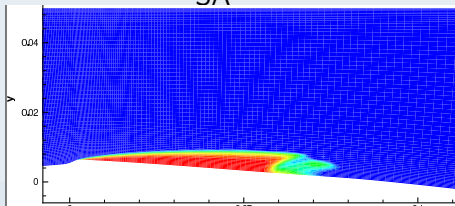


## Measurements

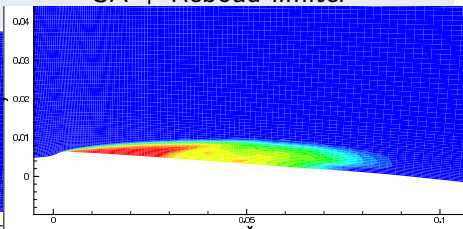
- Time-averaged longitudinal velocity and void ratio
- Time-averaged wall pressure evolution and RMS fluctuations

## Results using the Spalart-Allmaras model, density gradient

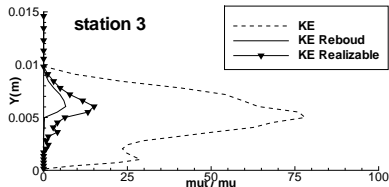
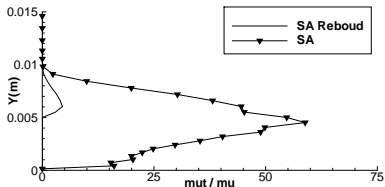
SA



SA + Rebound limiter

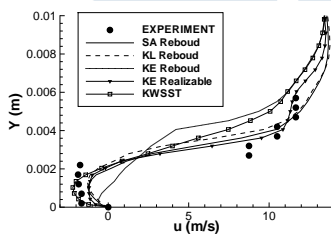
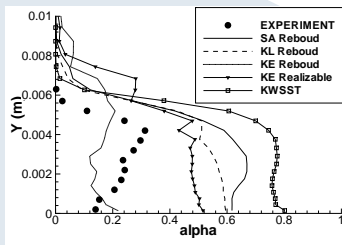
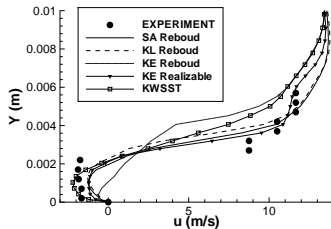
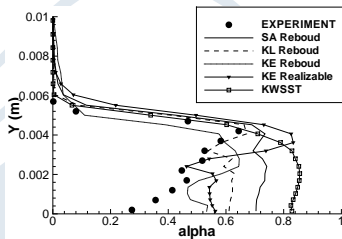


## Limitation of the eddy viscosity $\mu_t$



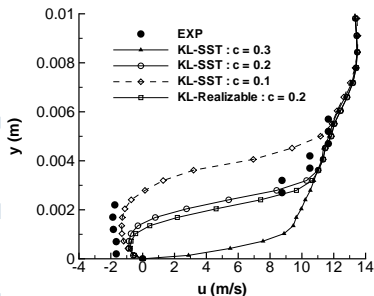
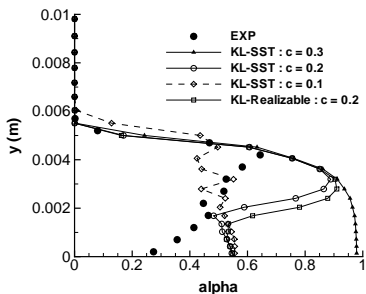


# Eddy viscosity limiter, profiles at stations 3 and 4

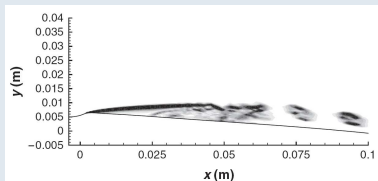
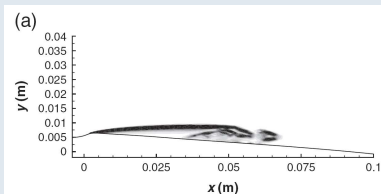


Time-averaged void fraction (left) and velocity (right) profiles

# The $k-l$ model + SST correction



Time-averaged void fraction (left) and velocity (right) profiles



Contour of the density gradient :  $a_1 = 0.2$  (left) and  $a_1 = 0.1$  (right)

# Compressibility corrections

## Pressure-dilatation, Sarkar formulation

$$\rho \Pi = \overline{P' \frac{\partial u_i''}{\partial x_i}} = -\alpha_2 \rho P M_t + \alpha_3 \rho \varepsilon_s M_t^2$$

$\alpha_2$ ,  $\alpha_3$  are constants to calibrate.  $M_t = \frac{\sqrt{k}}{2c}$  is the turbulent Mach number.

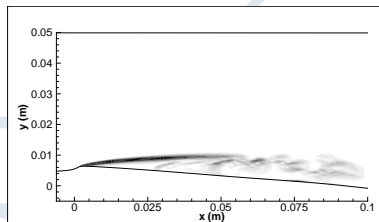
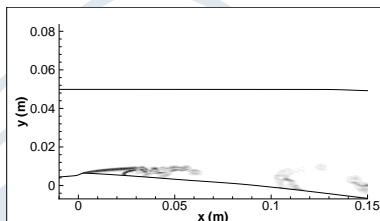
## Dilatational dissipation, Sarkar formulation

$$\varepsilon_d = \frac{4}{3} \mu \overline{s'_{kk} s'_{ll}} = \alpha_1 \varepsilon_s M_t^2 \quad \text{with } \alpha_1 \text{ to calibrate}$$

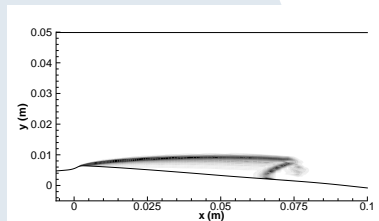
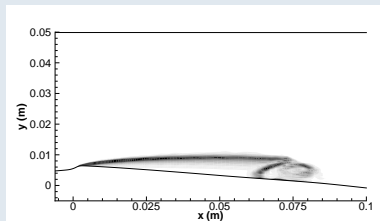
## Mass flux, Jones formulation

$$\rho M = \frac{\overline{\rho' u_i''}}{\rho} \left( \frac{\partial \bar{P}}{\partial x_i} - \frac{\partial \bar{\sigma}_{il}}{\partial x_l} \right) = -\frac{\mu_t}{\rho^2 \sigma_p} \frac{\partial \rho}{\partial x_i} \frac{\partial P}{\partial x_i}$$

$\sigma_p$  is a turbulent Schmidt number, which has to be calibrated.

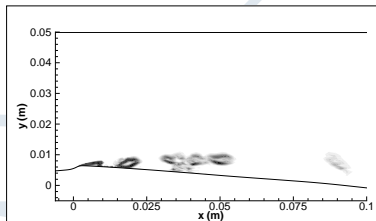
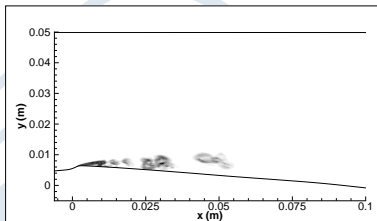


$k - \epsilon + \Pi$ ,  $\alpha_2=0.15$ ,  $\alpha_3 = 0.001$  (left) and  $\alpha_3 = 0.025$  (right)

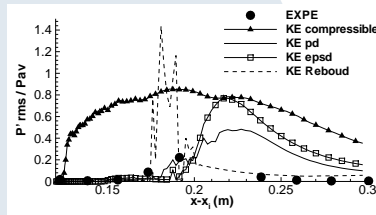
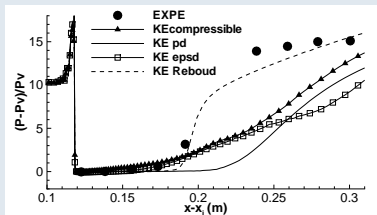


$k - \epsilon +$  mass flux term,  $\sigma_p = 1$  (left) and  $\sigma_p = 0.0001$  (right)

# $k - \epsilon$ compressible model (2)



$k - \epsilon + \Pi + M + \epsilon_d$ , Sarkar values, at 2 instants



Time-averaged wall pressure (left) and RMS fluctuations (right)



# Wall functions

## Opened questions

- Existence of an universal velocity profile.
- Instantaneous logarithmic area.
- Cavitating law of the wall ( $\kappa$  function of  $\alpha$ ).
- Turbulence damping functions.
- Modifications of turbulent properties downstream a pocket.
- Compressibility effects.

## Numerical study

- Computations using various meshes :  $y^+$  from 1 to 50.
- Comparison of wall functions : two-layer model versus TBLE model.



## Formulation - similar to single-phase flows

$$u^+ = y^+ \quad \text{if } y^+ < 11.13$$

$$u^+ = \frac{1}{\kappa} \ln y^+ + 5.25 \quad \text{if } y^+ > 11.13$$

where  $\kappa = 0.41$  is the von Karman constant

For unsteady flows, validity of the velocity profile at each instant.

Turbulent quantities : the production of  $k$  or directly  $k$  is fixed following the formulation by Viegas and Rubesin :

$$P_k = \frac{1}{y} \int_0^y \tau_{xy}^t \frac{\partial u}{\partial y} dy$$

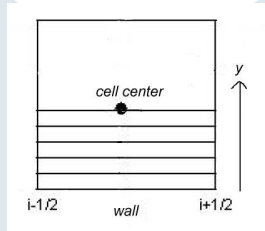
The second variable is computed through a length scale.

## Formulation

Simplified momentum equation :

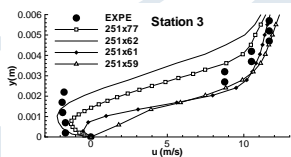
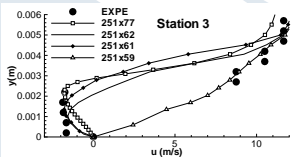
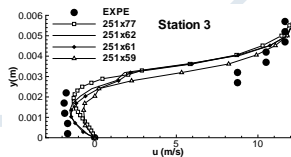
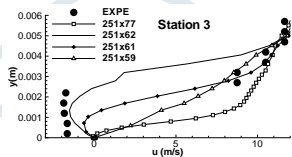
$$\frac{\partial u_i}{\partial t} + \frac{\partial u_i u_j}{\partial x_j} + \frac{1}{\rho} \frac{dP}{dx_i} = \frac{\partial}{\partial y} \left[ (\mu + \mu_t) \frac{\partial u_i}{\partial y} \right]$$

Use of an embedded grid between the first grid point and the wall.

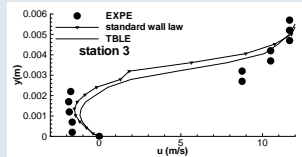
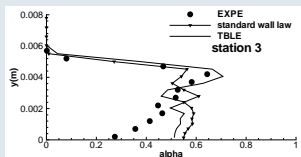


Discretization and integration of TBL equations in the embedded mesh. Iterative solving (Newton algorithm) for the variable  $\tau_w$ . The number of nodes in the embedded grid is  $N = 30$ .

# Venturi simulations



## Mesh influence near wall, station 3



## Two-layer model versus TBLE model

## Frequency and CPU cost

The experimental frequency is around 45 Hz.

The inlet cavitation parameter  $\sigma_{inlet} \sim 2.15$ .

mesh	$\sigma_{inlet}$	frequency (Hz)	cost for 100 ite. (s)	ratio
174 × 77	2.13	30	525.6	1
	2.18	no frequency		
174 × 62	2.13	35	429.6	0.817
	2.17	40		
174 × 60	2.13	35	419.4	0.80
	2.19	43.5		
174 × 59	2.14	44	408	0.776
174 × 57	2.145	46	393	0.747
174 × 56	2.14	46	387	0.736

# Improved model and hybrid RANS/LES

## Framework

Starting point : the  $k - kl$  model of Rotta (1972) where  $kl = \frac{3}{16} \int_{-\infty}^{\infty} R_{ii}(\vec{x}, r_y) dr_y$

Correlation tensor  $R_{ij}(\vec{x}, r_y) = \overline{U_i(\vec{x}) U_j(\vec{x} + r_y)}$

velocity correlation between a fixed point and a moving point in direction  $y$

Transport equation for  $\Psi = kl$  involving the integral quantity  $I$  :

$$I = -\frac{3}{16} \int_{-\infty}^{\infty} \frac{\partial U(\vec{x} + r_y)}{\partial y} R_{12} dr_y$$

Expansion in Taylor series gives :

$$I \approx \frac{\partial U(\vec{x})}{\partial y} \int_{-\infty}^{\infty} R_{12} dr_y + \frac{\partial^2 U(\vec{x})}{\partial y^2} \int_{-\infty}^{\infty} R_{12} r_y dr_y + \frac{1}{2} \frac{\partial^3 U(\vec{x})}{\partial y^3} \int_{-\infty}^{\infty} R_{12} r_y^2 dr_y + \dots$$

The second order derivative is neglected (assumption of homogeneous flow). The third order derivative is difficult to model and Rotta neglected also this term.

## Menter modelling

Menter proposed a model using the second derivative of the velocity (indicator of the heterogeneity of the flow) and the von Karman scale  $L_{vk}$

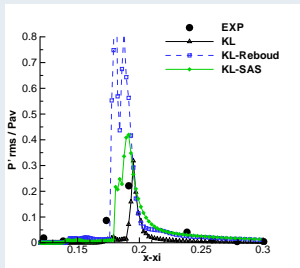
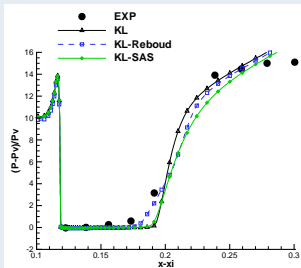
$$\frac{\partial^2 U}{\partial y^2} \int_{-\infty}^{\infty} R_{12} r_y dr_y \approx P_k \frac{k\ell}{k} \left( \frac{\ell}{L_{vk}} \right)^2 \quad \text{and} \quad L_{vk} = \kappa \left| \frac{U'}{U''} \right|$$

New term in the transport equation for  $\Psi$  (S-A term), driven by a constant  $\xi$ .  
 $\xi = 1.47$  following the calibration of Menter.

## Scale-adaptive model

- The characteristic length scale is self-adaptive, function of the von Karman scale for a standard model, the length scale is proportional to  $\delta$
- It allows to adjust the solve of turbulent structures  $\rightarrow$  behaviour close to a LES
- BUT : problem in the near-wall area, the log zone is not respected  
 $\rightarrow$  the S-A term is not activated in the near-wall area
- calibration of  $\xi$  in cavitating flows.

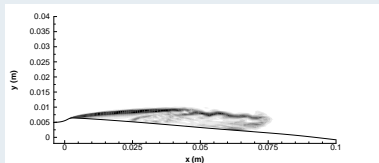
## Comparison between Rebound and S-A $k-l$ models



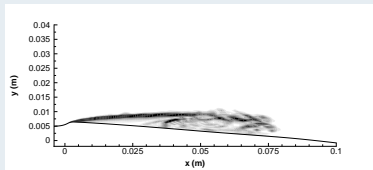
- time-averaged profiles quasi similar
- better pressure fluctuations with S-A model

## Density gradient, Schlieren visualization

$k-l$  SAS



$k-l$  Rebound





## Detached Eddy Simulation of Spalart

$$\frac{\partial \rho \tilde{\nu}}{\partial t} + \text{div} \left[ \vec{V} \rho \tilde{\nu} - \frac{1}{\sigma} (\mu + \rho \tilde{\nu}) g \vec{\text{rad}} \tilde{\nu} \right] = c_{b1} (1 - f_{t2}) \tilde{S} \rho \tilde{\nu} + \frac{c_{b2}}{\sigma} g \vec{\text{rad}} \rho \tilde{\nu} \cdot g \vec{\text{rad}} \tilde{\nu} - \left( c_{\omega 1} f_{\omega} - \frac{c_{b1}}{\kappa^2} f_{t2} \right) \rho \frac{\tilde{\nu}^2}{\tilde{d}^2}$$

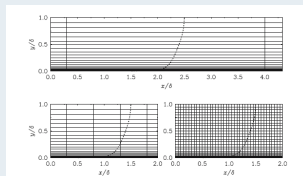
with  $\tilde{d} = \min(d, C_{DES} \Delta)$  and  $\Delta = \max(\Delta x, \Delta y, \Delta z)$ .

$C_{DES}$  is a constant evaluated for the decay of isotropic turbulence = 0.65

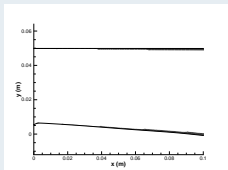
In equilibrium area :  $\tilde{\nu} = C_{DES}^2 \Delta^2 S$

## Drawbacks

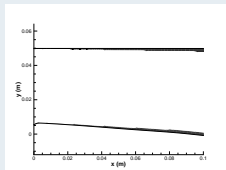
- Grid induced separation.
- Transition between RANS-mode and LES-mode : "grey" zone.
- Calibration of the constant  $C_{des}$ .



## Influence of $C_{DES}$ . RANS regions (black) and LES (white)

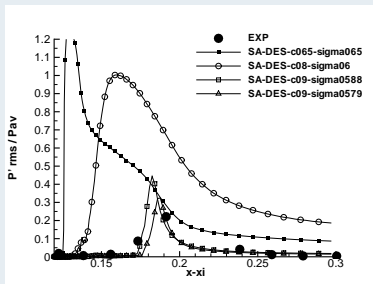
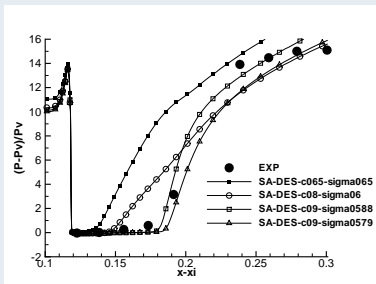


$C_{des} = 0.65$

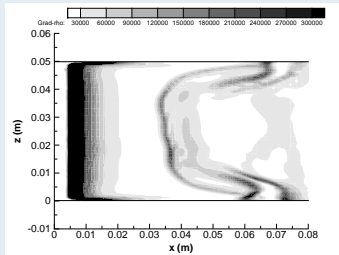
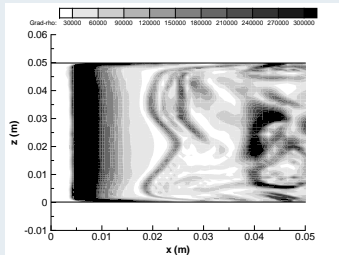
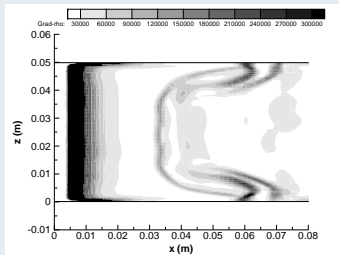
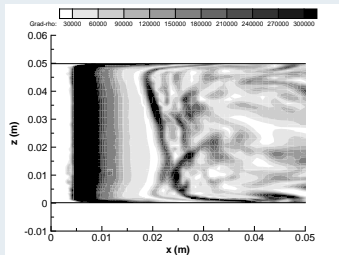


$C_{des} = 0.9$

## Wall pressure and RMS fluctuations

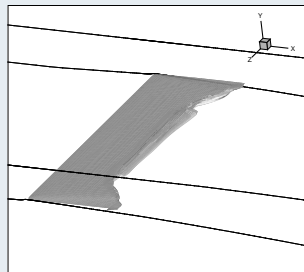
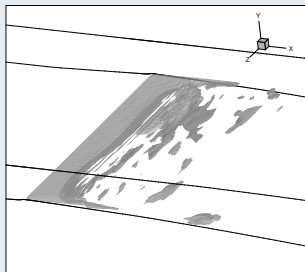
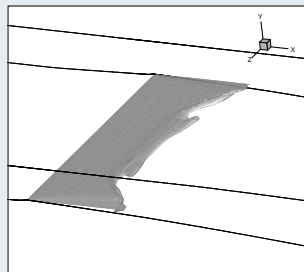
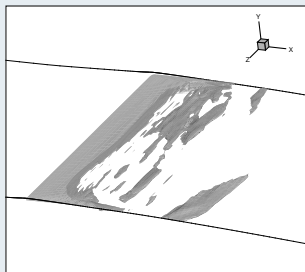


## Comparison between $k - \ell$ S-A and DES simulations



gradient density visualization at two different instants :  $k - \ell$  S-A (left) and DES (right)

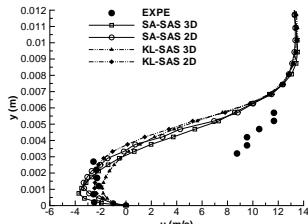
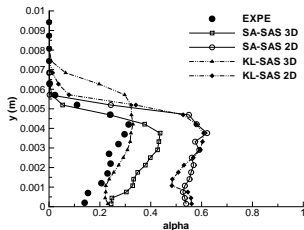
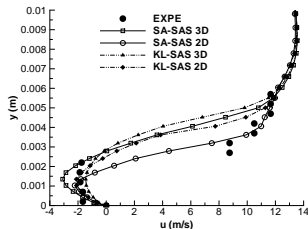
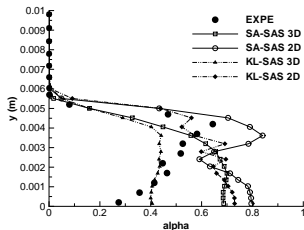
## Comparison between $k - \ell$ Scale-Adaptive and DES simulations



Iso-surface of the void fraction for the value of 60% at two different

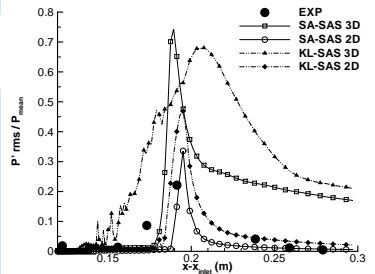
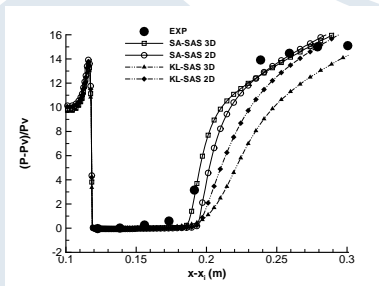
## Comparison between 2D and 3D simulations

- Good agreement between models and experiment (mid-span profiles)
- 3D computations provides void fraction lower than 2D computations



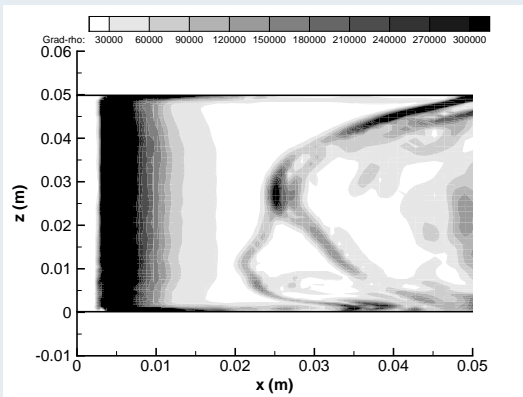
## Comparison between 2D and 3D simulations

The level of RMS pressure fluctuations is largely overestimated by the 3D computations



Time-averaged wall pressure and RMS fluctuations

## Dynamic behaviour - oblique mode using S-A simulations



Density gradient visualization

→ a transversal instability at a low frequency : 6 Hz.

# One-fluid filtered equations and LES



## Scales

- Micro-scales, scales which are small enough to describe individual bubble shapes.
- Meso-scales, which are comparable to bubble sizes.
- Macro-scales, which entail enough bubbles for statistical representation.
- When LES is applied at a micro-scale, combination with interface tracking methods (see Lakehal).
- When LES is applied at a macro-scale, the interface resolution is not considered.
- The scale separation is mathematically obtained by applying a convolution product using a large-scale-pass filter (function  $G$ ).

## Filter

- For a quantity  $\phi$ , the filtered variable is defined as :  $\overline{\phi} = G \circ \phi$
- The Favre filtered variable :  $\tilde{\phi} = \overline{\rho\phi}/\overline{\rho}$
- The filtered phase indicator function  $\overline{X_k} = G \circ X_k = \alpha_k$  can be interpreted as a filtered volume fraction of phase  $k$ .

## Phase indicator function

The phase indicator function is defined as :

$$X_k(M(x, t)) = \begin{cases} 1 & \text{if phase } k \text{ is present in point } M(x, t) \text{ at } t \\ 0 & \text{otherwise} \end{cases}$$

## Equations

Assumptions : the filtering operator commutes with time and spatial derivatives. The mass transfer is assuming to be proportional to the velocity divergence through a constant  $C$ .

$$\begin{aligned} \frac{\partial \bar{\rho}}{\partial t} + \frac{\partial (\bar{\rho} \tilde{u}_i)}{\partial x_j} &= 0 \\ \frac{\partial \bar{\rho} \tilde{u}_i}{\partial t} + \frac{\partial (\bar{\rho} \tilde{u}_i \tilde{u}_j + \bar{P} \delta_{ij})}{\partial x_j} &= \text{div} \left( 2\bar{\mu} \tilde{S} + 2\tau_{\mu S} - 2\tau_{\rho S} - \tau_{\rho uu} \right) \\ \frac{\partial \bar{\rho} \tilde{E}}{\partial t} + \frac{\partial (\bar{\rho} \tilde{H} \tilde{u}_i + \bar{Q}_i^v)}{\partial x_j} &= \text{div} \left( 2\tau_{\mu S u} - 2\bar{\mu} \tilde{V} \tau_{\rho S} - 2\bar{\mu} \tilde{S} \tau_{\rho u} \right) \\ &+ \text{div} \left( 4\bar{\mu} \tau_{\rho u} \tau_{\rho S} + \bar{\rho} \tilde{H} \tau_{\rho u} - \tau_{\rho H u} \right) \\ \frac{\partial \alpha}{\partial t} + \tilde{V} \cdot \nabla \alpha - C \frac{\partial \tilde{u}_i}{\partial x_j} &= -\tau_{u\alpha} - C \text{div} (\tau_{\rho u}) \end{aligned}$$

## Subgrid terms

$$\begin{aligned}
 \tau_{u\alpha} &= \overline{V \cdot \nabla X_v} - \tilde{V} \cdot \nabla \alpha & ; & & \tau_{\rho u} &= \tilde{V} - \overline{V} \\
 \tau_{\mu S} &= \overline{\mu S} - \bar{\mu} \bar{S} & ; & & \tau_{\rho S} &= \tilde{S} - \bar{S} \\
 \tau_{\rho H u} &= \overline{\rho H V} - \bar{\rho} \tilde{H} \tilde{V} & ; & & \tau_{\mu S u} &= \overline{\mu S V} - \bar{\mu} \bar{S} \bar{V} \\
 \tau_{\rho u u} &= \bar{\rho} \left( \widetilde{V \otimes V} - \tilde{V} \otimes \tilde{V} \right)
 \end{aligned}$$

- The subgrid term  $\tau_{u\alpha}$  is specific to two-phase flows.
- The influence and the hierarchy of all these terms have never been investigated in cavitating flows.
- The magnitude of the different subgrid terms was *a priori* evaluated in the case of phase separation flows and turbulence bubble interaction (Labourasse, Vincent)  $\rightarrow$  the influence of  $\tau_{u\alpha}$  is highly dependent on the flow configurations and/or on the chosen two-phase description.

## Modelling difficulties

- Lots of assumption in models.
- Lack of experimental data or DNS.
- Compressibility turbulent closure due to the no divergence free fluctuating velocity field
- Advanced turbulence models and hybrid turbulence model to improve the level of resolved scales
- A challenge : LES in cavitating flows.