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Turbulence modelling in cavitating flows

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LABORATOIRE DES ECOULEMENTS
GÉOPHYSIQUES ET INDUSTRIELS

Outline

- 1 General considerations
- 2 One-fluid RANS equations
- 3 Limitation of eddy viscosity
- 4 Compressibility terms
- 5 Wall models
- 6 Improved models - hybrid RANS/LES
- 7 One-fluid filtered equations and LES
- 8 Conclusions

General considerations

Questions

- Kolmogorov spectrum, slope $\neq -5/3$
- Kolmogorov scale / size of two-phase structures.
- Induced turbulence or pseudo-turbulence.
- Compressibility effects on turbulence.
- Anisotropy of the Reynolds tensor.
- Increase or decrease of the turbulence intensity.
- Cavitation-turbulence interaction at small scales.

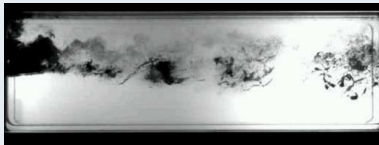
Remarks

- no DNS data or turbulent quantities measurements.
- non universality of two-phase flows.

Cavitation pockets



Cavitation in vortex



Turbulence scales

- DNS : all scales are resolved
- LES : large scale are resolved and small scales are modelled
- RANS : mean flow is resolved and turbulence is modelled
- Hybrid RANS/LES : model adapts to the mesh resolution

Phase scales

- DNS : resolution of each fluid + interface
- pseudo-DNS : resolution of each fluid + tracking of the interface (VoF, level set...)
- filtered (LES) and averaged (RANS) models :
 - two-fluid model,
 - reduced models,
 - one-fluid model

Averaged one-fluid equations (RANS)

- Temporal averaged or ensemble averaged equations
- The same operator for two-phase structure and turbulent structure
- Boussinesq analogy used similarly to single-phase flows
- Transport-equation models : $k - \varepsilon$, $k - \omega$,...

Filtered one-fluid equations (LES)

- First works in incompressible flow without phase transition
- Lots of sub-grid terms - problem of modelling
- Link with the DNS

Averaged equations

Temporal phase average

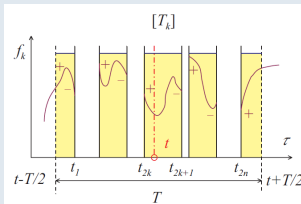
Averaging over a time T :

$$\overline{\phi}_k = \frac{1}{T} \int_T \phi_k(x, \tau) d\tau$$

Averaging over the time T_k of presence of the phase k :

$$\overline{\overline{\phi}}_k = \frac{1}{T_k} \int_{T_k} \phi_k(x, \tau) d\tau$$

$$\text{Void fraction : } \alpha(x, t) = \frac{T_k}{T} = \frac{\overline{\overline{\phi}}_k}{\overline{\phi}_k}$$



Mass-weighted average (Favre average)

For a variable $\rho\phi$, a mass-weighted average :

$$\tilde{\phi}_k = \frac{\overline{\rho_k \phi_k}}{\overline{\rho_k}} = \frac{\overline{\overline{\rho_k \phi_k}}}{\overline{\overline{\rho_k}}} \quad \text{and} \quad \tilde{\phi} = \frac{\sum \alpha_k \overline{\overline{\rho_k \phi_k}}}{\sum \alpha_k \overline{\overline{\rho_k}}} = \frac{\sum \overline{\rho_k \phi_k}}{\sum \overline{\rho_k}}$$

Mass-weighted decomposition

An average part and a fluctuation (two-phase and turbulent contribution) :

$$\begin{aligned} \rho_k &= \overline{\overline{\rho_k}} + \rho'_k, & u_k &= \tilde{u}_k + u''_k, & \phi_k &= \tilde{\phi}_k + \phi''_k \\ \overline{\overline{\rho'_k}} &= 0, & \overline{\overline{\rho_k u''_k}} &= 0, & \overline{\overline{\rho_k \phi''_k}} &= 0 \end{aligned}$$

Conservative equations for the phase k

Mass conservation :

$$\frac{\partial \alpha_k \bar{\rho}_k}{\partial t} + \text{div} (\alpha_k \bar{\rho}_k \tilde{u}_k) = \Gamma_k$$

Momentum conservation :

$$\frac{\partial \alpha_k \overline{\rho_k \tilde{u}_k}}{\partial t} + \text{div} (\alpha_k \bar{\rho}_k \tilde{u}_k \otimes \tilde{u}_k + P_k) = \text{div} (\alpha_k (\bar{\tau}_k + \tau_k^t)) + \mathbb{M}_k$$

Total energy conservation :

$$\begin{aligned} \frac{\partial \alpha_k \bar{\rho}_k (\tilde{E}_k + k_k)}{\partial t} + \text{div} (\alpha_k \rho_k (\tilde{E}_k + k_k) \tilde{u}_k) &= \text{div} (\alpha_k P_k \tilde{u}_k) + \text{div} (\alpha_k (\bar{\tau}_k + \tau_k^t) \tilde{u}_k) \\ &\quad - \text{div} (\alpha_k (\bar{q}_k + q_k^t)) + \mathbb{Q}_k \end{aligned}$$

New unknowns : Reynolds stress tensor τ_k^t and turbulent heat flux q_k^t

Fluctuating fields

The velocity fluctuating field is not divergence free (even with incompressible phases) :

$$\frac{\partial u'_{k,l}}{\partial x_l} = -\frac{\partial \bar{u}_{k,l}}{\partial x_l} = \frac{1}{\alpha_k} u'_k \cdot n_k \delta_l$$

It leads to supplementary unknowns, called "compressible terms".

Equation for the momentum fluctuation :

$$\begin{aligned} \rho_k \frac{\partial u''_{k,i}}{\partial t} + \rho_k \tilde{u}_{k,l} \frac{\partial u''_{k,i}}{\partial x_l} &+ \rho_k u''_{k,l} \frac{\partial \tilde{u}_{k,i}}{\partial x_l} + \rho_k u_{k,l} \frac{\partial u''_{k,i}}{\partial t} \\ &= \frac{\partial \mathbb{T}_{k,il}}{\partial x_l} + \rho_k \tilde{u}_{k,i} \frac{\partial \tilde{u}_{k,l}}{\partial x_l} \\ &+ \frac{\rho_k}{\bar{\rho}_k \alpha_k} \left[-\frac{\partial \alpha_k \bar{\mathbb{T}}_{k,il}}{\partial x_l} - \mathbb{M}_k + \frac{\partial}{\partial x_l} \left(\alpha_k \bar{\rho}_k \widetilde{u''_{k,i} u''_{k,l}} \right) \right] \\ &+ \frac{\rho_k}{\bar{\rho}_k \alpha_k} \left[\tilde{u}_{k,i} \left(\frac{\partial \alpha_k \bar{\rho}_k}{\partial t} + \tilde{u}_{k,l} \frac{\partial \alpha_k \bar{\rho}_k}{\partial x_l} \right) \right] \end{aligned}$$

Turbulent kinetic energy equation for the phase k

it is possible to write the equation for the phasic Reynolds stress and the phasic TKE k_k

$$\frac{\partial \alpha_k \bar{\rho}_k k_k}{\partial t} + \text{div} (\alpha_k \bar{\rho}_k k_k \tilde{u}_k) = \alpha_k \bar{\rho}_k (P_k - \epsilon_k + \Pi_k + M_k + D_k) + \Gamma_k K^\Gamma$$

with :

$$\bar{\rho}_k \alpha_k P_k = -\alpha_k \bar{\rho}_k \widetilde{u''_{k,i} u''_{k,l}} \frac{\partial \tilde{u}_{k,i}}{\partial x_l} \quad \text{Production term}$$

$$\bar{\rho}_k \alpha_k \epsilon_k = \overline{\alpha_k \tau'_{k,il} \frac{\partial u''_{k,i}}{\partial x_l}} \quad \text{Dissipation rate}$$

$$\bar{\rho}_k \alpha_k \Pi_k = \overline{\alpha_k p'_k \frac{\partial u''_{k,i}}{\partial x_i}} \quad \text{Pressure-dilatation term}$$

$$\bar{\rho}_k \alpha_k M_k = \overline{\alpha_k u''_{k,i} \left[-\frac{\partial \bar{p}_k}{\partial x_i} + \frac{\partial \bar{\sigma}_{k,il}}{\partial x_l} \right]} \quad \text{Mass flux term}$$

$$\bar{\rho}_k \alpha_k D_k = -\frac{\partial}{\partial x_l} \left[\overline{\alpha_k P'_k u''_{k,i}} \delta_{il} - \overline{\alpha_k \tau'_{k,il} u''_{k,i}} \right] - \frac{\overline{\alpha_k \rho_k \frac{u''_{k,i} u''_{k,i}}{2} u''_{k,l}}}{\partial x_l} \quad \text{Diffusion term}$$

$$\Gamma_k K^\Gamma \quad \text{Mass transfer term}$$

Mixture mean quantities

- mixture density and pressure : $\rho_m = \sum \alpha_k \bar{\rho}_k$, $P_m = \sum \alpha_k \bar{P}_k$
- mixture internal energy : $\rho_m e_m = \sum \alpha_k \bar{\rho}_k \tilde{e}_k$
- mass center velocity : $\rho_m u_{m,i} = \sum \alpha_k \bar{\rho}_k \tilde{u}_{k,i}$
- mixture viscosity : $\mu_m = \sum \alpha_k \bar{\mu}_k$
- mixture viscous stress tensor : $\tau_{m,ij} = \sum \alpha_k \overline{\tau_{k,ij}}$
- mixture heat flux : $q_m = \sum \alpha_k \bar{q}_k$

Mixture turbulent quantities

- mixture turbulent kinetic energy : $k_m = \sum \alpha_k k_k = \sum \alpha_k \widetilde{u_{k,i}''^2} / 2$
- mixture Reynolds stress tensor : $\tau_{m,ij}^t = - \sum \alpha_k \overline{\rho_k u_{k,i}'' u_{k,j}''}$
- mixture eddy viscosity : $\mu_{tm} = \sum \alpha_k \mu_{tk}$
- mixture turbulent heat flux : $q_m^t = \sum \alpha_k q_k^t$
- mixture dissipation rate : ε_m , mixture specific dissipation $\omega_m \dots$

Conservative equations

Mass equation :

$$\frac{\partial \rho_m}{\partial t} + \operatorname{div}(\rho_m u_m) = 0$$

Momentum equation :

$$\frac{\partial \rho_m u_m}{\partial t} + \operatorname{div}(\rho_m u_m \otimes u_m + P_m) = \operatorname{div}(\tau_m + \tau_m^t)$$

Energy equation :

$$\begin{aligned} \frac{\partial \rho_m (E_m + k_m)}{\partial t} + \operatorname{div}(\rho_m (E_m + k_m) u_m) &= \operatorname{div}(-P_m u_m) - \operatorname{div}(q_m - q_m^t) \\ &+ \operatorname{div}[(\tau_m + \tau_m^t) u_m] \end{aligned}$$

Mixture turbulent kinetic energy equation

$$\frac{\partial \rho_m k_m}{\partial t} + \text{div}(\rho_m k_m u_m) = \rho_m P_m + \rho_m \Pi_m - \rho_m \epsilon_m + \rho_m M_m + \rho_m D_m + \Gamma_m K_m^\Gamma$$

It is assumed that each phase shares the same fluctuating velocity u_i'' , the same fluctuating pressure P' and the same fluctuating viscous stress τ'_{ij} :

$$\begin{aligned} \rho_m P_m &= \tau_{m,il}^t \frac{\partial u_{m,i}}{\partial x_l} & ; & & \rho_m \Pi_m &= \overline{P' \frac{\partial u_i''}{\partial x_i}} \\ \rho_m \epsilon_m &= \overline{\tau'_{il} \frac{\partial u_i''}{\partial x_l}} & ; & & \Gamma_m K_m^\Gamma &= 0 \end{aligned}$$

$$\rho_m M_m = \overline{u_i''} \left[-\frac{\partial \rho_m}{\partial x_i} \right] + \overline{u_i''} \left[\frac{\partial \sigma_{m,il}}{\partial x_l} - \sum_k \overline{\overline{\sigma}}_{k,il} \frac{\partial \alpha_k}{\partial x_l} \right]$$

$$\rho_m D_m = -\frac{\partial}{\partial x_l} \left[\overline{\rho' u_i''} \delta_{il} - \overline{\sigma'_{il} u_i''} \right] - \frac{\partial \rho_m \overline{\frac{u_i'' u_i''}{2} u_l''}}{\partial x_l}$$

Main features

- A fluctuation is associated to a mixture quantity !
- The fluctuating velocity field is not divergence free \rightarrow supplementary terms, difficult to model.
- The pressure-dilatation term $\rho_m \Pi_m$ is null in mean but not instantaneously.
- The mixture dissipation ε is not only solenoidal.
Inhomogeneous and dilatational (or compressible) contributions.

$$\rho \varepsilon \approx \underbrace{2\mu \overline{\omega'_{ik} \omega'_{ik}}}_{\rho \varepsilon_s} + \underbrace{2\mu \frac{\partial}{\partial x_k} \left[\frac{\partial \overline{u'_k u'_l}}{\partial x_l} - 2 \overline{u'_k s'_{ll}} \right]}_{\rho \varepsilon_{inh}} + \underbrace{\frac{4}{3} \mu \overline{s'_{kk} s'_{ll}}}_{\rho \varepsilon_d}$$

- The diffusion term is modelled with a gradient formulation.
- The mixture dissipation equation ε is completely modelled following the single-phase formulation.

Boussinesq assumption

Boussinesq analogy and mixture eddy viscosity assumption μ_{tm} :

$$\tau_{m,ij}^t = \mu_{tm} \left[\frac{\partial u_{m,i}}{\partial x_j} + \frac{\partial u_{m,j}}{\partial x_i} - \frac{2}{3} \text{div } u_m \delta_{ij} \right] - \frac{2}{3} \rho_m k_m \delta_{ij}$$

Evaluation of the mixture eddy viscosity with transport-equation models.

Turbulent Fourier law

Fourier law analogy and mixture thermal conductivity λ_m^t :

$$q_m^t = -\lambda_m^t \text{grad } T_m$$

Assumption of constant turbulent Prandtl number Pr_t :

$$\lambda_m^t = \sum \alpha_k \lambda_k^t = \sum \alpha_k \frac{\mu_k^t C_{pk}}{Pr_t} \quad \text{approximated by} \quad \lambda_m^t \simeq \frac{\mu_m^t C_{pm}}{Pr_t}$$

Usual model, $k - \varepsilon$ model for a mixture

transport-equation models equivalent to single-phase turbulent models.
all supplementary terms are neglected.
only the solenoidal dissipation is taken into account.

$$\begin{aligned}\frac{\partial \rho k}{\partial t} + \operatorname{div} \left[\rho u k - \left(\mu + \frac{\mu_t}{\sigma_k} \right) \operatorname{grad} k \right] &= \rho P_k - \rho \varepsilon \\ \frac{\partial \rho \varepsilon}{\partial t} + \operatorname{div} \left[\rho u \varepsilon - \left(\mu + \frac{\mu_t}{\sigma_\varepsilon} \right) \operatorname{grad} \varepsilon \right] &= c_{\varepsilon 1} \frac{\varepsilon}{k} \rho P_k - \rho c_{\varepsilon 2} f_2 \frac{\varepsilon^2}{k} \\ \mu_t &\approx \rho \frac{k^2}{\varepsilon_s}\end{aligned}$$

Remarks

- a large quantity of assumptions.
- introduction of wall treatment (damping functions or wall functions)
- a large quantity of problems !



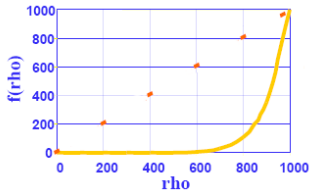
Limitation of the eddy viscosity

The Rebound correction

$$\mu_t = f(\rho) C_\mu \frac{k^2}{\epsilon}$$

$$f(\rho) = \rho_v + \left(\frac{\rho_v - \rho}{\rho_v - \rho_l} \right)^n (\rho_l - \rho_v)$$

n is usually set to 10



SST correction - Bradshaw's assumption for 2D boundary layer

$$\mu_t = \frac{\rho k / \omega}{\max\left(1, \frac{\Omega F_2}{a_1 \omega}\right)} ; a_1 = 0.3 ; \Omega = \sqrt{2\Omega_{ij}\Omega_{ij}} \text{ with } \Omega_{ij} = \frac{1}{2} \left(\frac{\partial \tilde{u}_i}{\partial x_j} - \frac{\partial \tilde{u}_j}{\partial x_i} \right)$$

Realisability constrains of Durbin

$$\mu_t = \min \left(C_\mu^0 ; \frac{c}{s\sqrt{3}} \right) \frac{\rho k^2}{\epsilon} ; 0 \leq c \leq 1 ; C_\mu^0 = 0.09 ; s = \frac{k}{\epsilon} S \text{ with } S = 2S_{ij}S_{ij} - \frac{2}{3}S_{kk}^2$$

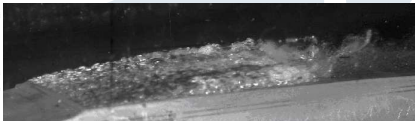
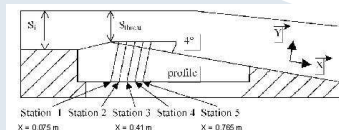
Experimental conditions

Operating point :

- $U_{inlet} = 10.8 \text{ m/s}$
- $\sigma_{inlet} = \frac{P_{inlet} - P_{vap}}{0.5 \rho U_{inlet}^2} \approx 0.55$

Observations :

- A quasi stable cavitation of 0.70 to 0.85 m length
- An unsteady closure region with vapour cloud shedding and a liquid re-entrant jet

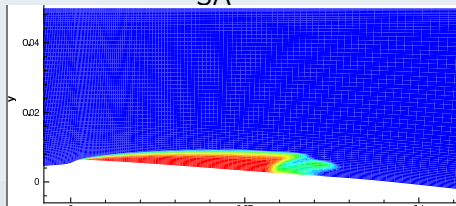


Measurements

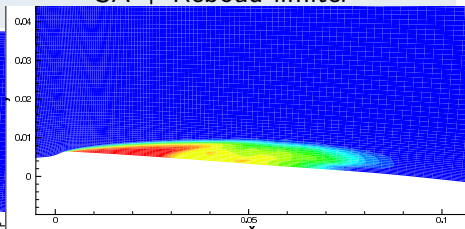
- Time-averaged longitudinal velocity and void ratio
- Time-averaged wall pressure evolution and RMS fluctuations

Results using the Spalart-Allmaras model, density gradient

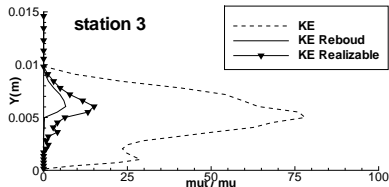
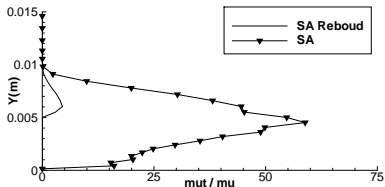
SA



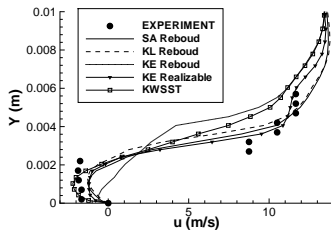
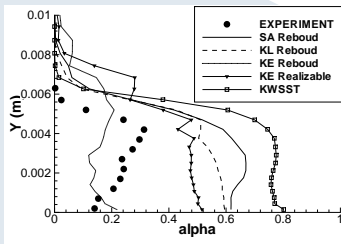
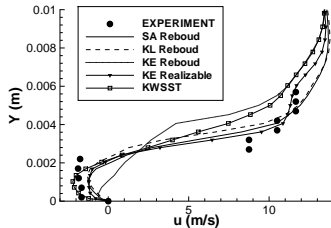
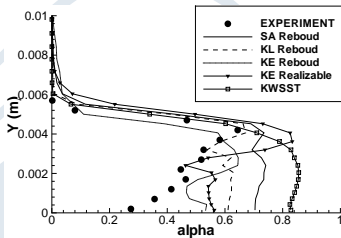
SA + Rebound limiter



Limitation of the eddy viscosity μ_t

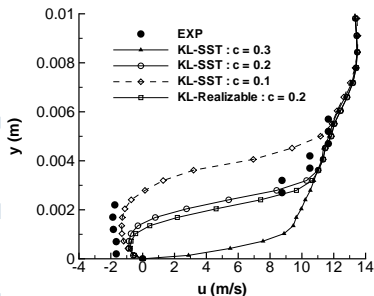
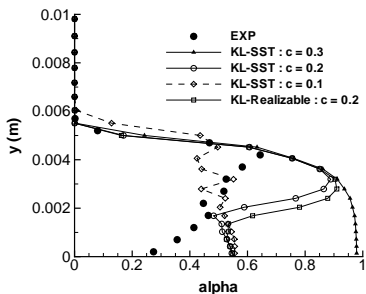


Eddy viscosity limiter, profiles at stations 3 and 4

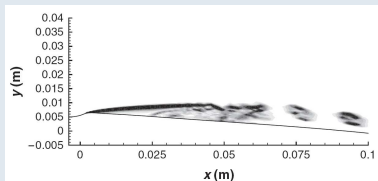
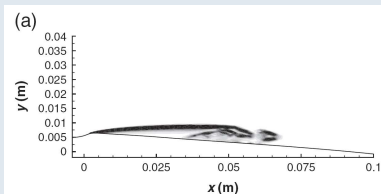


Time-averaged void fraction (left) and velocity (right) profiles

The $k-l$ model + SST correction



Time-averaged void fraction (left) and velocity (right) profiles



Contour of the density gradient : $a_1 = 0.2$ (left) and $a_1 = 0.1$ (right)

Compressibility corrections

Pressure-dilatation, Sarkar formulation

$$\rho \Pi = \overline{P' \frac{\partial u_i''}{\partial x_i}} = -\alpha_2 \rho P M_t + \alpha_3 \rho \varepsilon_s M_t^2$$

α_2, α_3 are constants to calibrate. $M_t = \frac{\sqrt{k}}{2c}$ is the turbulent Mach number.

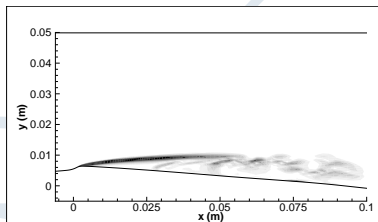
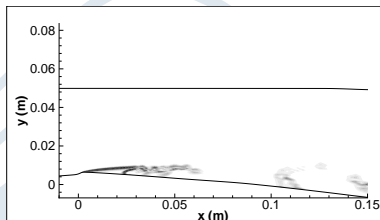
Dilatational dissipation, Sarkar formulation

$$\varepsilon_d = \frac{4}{3} \mu \overline{s'_{kk} s'_{ll}} = \alpha_1 \varepsilon_s M_t^2 \quad \text{with } \alpha_1 \text{ to calibrate}$$

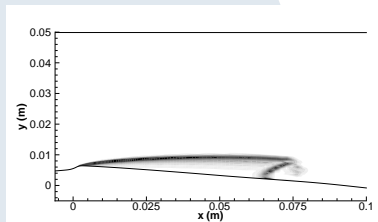
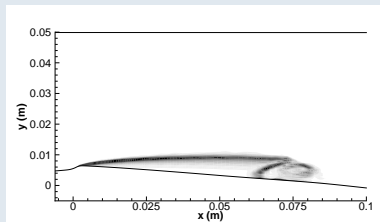
Mass flux, Jones formulation

$$\rho M = \frac{\overline{\rho' u_i''}}{\rho} \left(\frac{\partial \bar{P}}{\partial x_i} - \frac{\partial \bar{\sigma}_{il}}{\partial x_l} \right) = -\frac{\mu_t}{\rho^2 \sigma_p} \frac{\partial \rho}{\partial x_i} \frac{\partial P}{\partial x_i}$$

σ_p is a turbulent Schmidt number, which has to be calibrated.

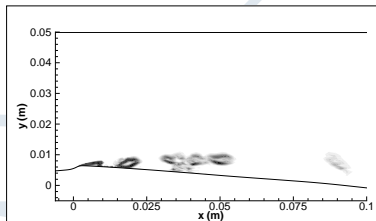
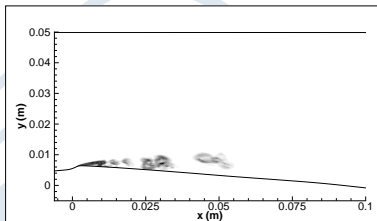


$k - \epsilon + \Pi$, $\alpha_2=0.15$, $\alpha_3 = 0.001$ (left) and $\alpha_3 = 0.025$ (right)

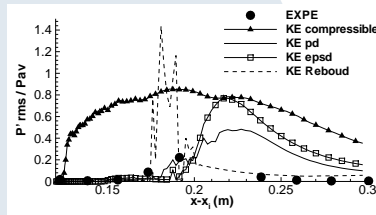
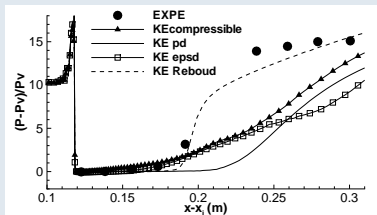


$k - \epsilon +$ mass flux term, $\sigma_p = 1$ (left) and $\sigma_p = 0.0001$ (right)

$k - \epsilon$ compressible model (2)



$k - \epsilon + \Pi + M + \epsilon_d$, Sarkar values, at 2 instants



Time-averaged wall pressure (left) and RMS fluctuations (right)



Wall functions

Opened questions

- Existence of an universal velocity profile.
- Instantaneous logarithmic area.
- Cavitating law of the wall (κ function of α).
- Turbulence damping functions.
- Modifications of turbulent properties downstream a pocket.
- Compressibility effects.

Numerical study

- Computations using various meshes : y^+ from 1 to 50.
- Comparison of wall functions : two-layer model versus TBLE model.

Formulation - similar to single-phase flows

$$u^+ = y^+ \quad \text{if } y^+ < 11.13$$

$$u^+ = \frac{1}{\kappa} \ln y^+ + 5.25 \quad \text{if } y^+ > 11.13$$

where $\kappa = 0.41$ is the von Karman constant

For unsteady flows, validity of the velocity profile at each instant.

Turbulent quantities : the production of k or directly k is fixed following the formulation by Viegas and Rubesin :

$$P_k = \frac{1}{y} \int_0^y \tau_{xy}^t \frac{\partial u}{\partial y} dy$$

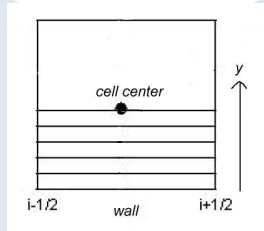
The second variable is computed through a length scale.

Formulation

Simplified momentum equation :

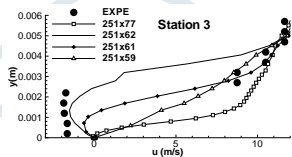
$$\frac{\partial u_i}{\partial t} + \frac{\partial u_i u_j}{\partial x_j} + \frac{1}{\rho} \frac{dP}{dx_i} = \frac{\partial}{\partial y} \left[(\mu + \mu_t) \frac{\partial u_i}{\partial y} \right]$$

Use of an embedded grid between the first grid point and the wall.

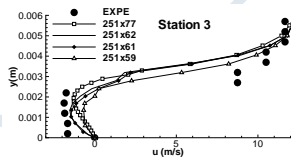


Discretization and integration of TBL equations in the embedded mesh. Iterative solving (Newton algorithm) for the variable τ_w . The number of nodes in the embedded grid is $N = 30$.

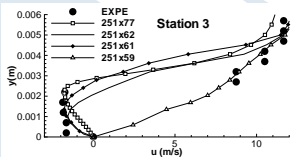
Venturi simulations



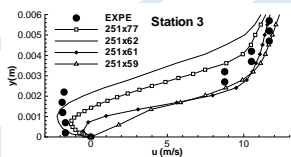
SA



KL

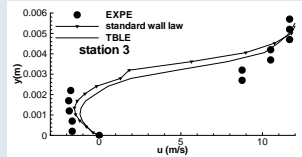
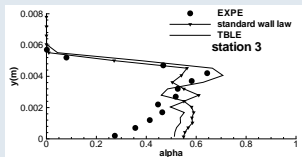


KE



KWSST

Mesh influence near wall, station 3



Two-layer model versus TBLE model

Frequency and CPU cost

The experimental frequency is around 45 Hz.

The inlet cavitation parameter $\sigma_{inlet} \sim 2.15$.

mesh	σ_{inlet}	frequency (Hz)	cost for 100 ite. (s)	ratio
174 × 77	2.13	30	525.6	1
	2.18	no frequency		
174 × 62	2.13	35	429.6	0.817
	2.17	40		
174 × 60	2.13	35	419.4	0.80
	2.19	43.5		
174 × 59	2.14	44	408	0.776
174 × 57	2.145	46	393	0.747
174 × 56	2.14	46	387	0.736

Improved model and hybrid RANS/LES

Framework

Starting point : the $k - kl$ model of Rotta (1972) where $kl = \frac{3}{16} \int_{-\infty}^{\infty} R_{ii}(\vec{x}, r_y) dr_y$

Correlation tensor $R_{ij}(\vec{x}, r_y) = \overline{U_i(\vec{x}) U_j(\vec{x} + r_y)}$

velocity correlation between a fixed point and a moving point in direction y

Transport equation for $\Psi = kl$ involving the integral quantity I :

$$I = -\frac{3}{16} \int_{-\infty}^{\infty} \frac{\partial U(\vec{x} + r_y)}{\partial y} R_{12} dr_y$$

Expansion in Taylor series gives :

$$I \approx \frac{\partial U(\vec{x})}{\partial y} \int_{-\infty}^{\infty} R_{12} dr_y + \frac{\partial^2 U(\vec{x})}{\partial y^2} \int_{-\infty}^{\infty} R_{12} r_y dr_y + \frac{1}{2} \frac{\partial^3 U(\vec{x})}{\partial y^3} \int_{-\infty}^{\infty} R_{12} r_y^2 dr_y + \dots$$

The second order derivative is neglected (assumption of homogeneous flow). The third order derivative is difficult to model and Rotta neglected also this term.

Menter modelling

Menter proposed a model using the second derivative of the velocity (indicator of the heterogeneity of the flow) and the von Karman scale L_{vk}

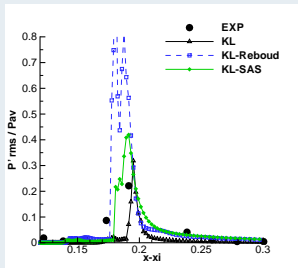
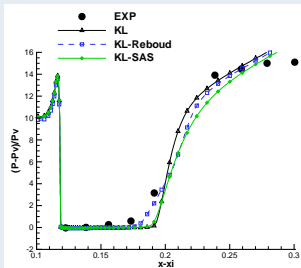
$$\frac{\partial^2 U}{\partial y^2} \int_{-\infty}^{\infty} R_{12} r_y dr_y \approx P_k \frac{k\ell}{k} \left(\frac{\ell}{L_{vk}} \right)^2 \quad \text{and} \quad L_{vk} = \kappa \left| \frac{U'}{U''} \right|$$

New term in the transport equation for Ψ (S-A term), driven by a constant ξ .
 $\xi = 1.47$ following the calibration of Menter.

Scale-adaptive model

- The characteristic length scale is self-adaptive, function of the von Karman scale for a standard model, the length scale is proportional to δ
- It allows to adjust the solve of turbulent structures \rightarrow behaviour close to a LES
- BUT : problem in the near-wall area, the log zone is not respected
 \rightarrow the S-A term is not activated in the near-wall area
- calibration of ξ in cavitating flows.

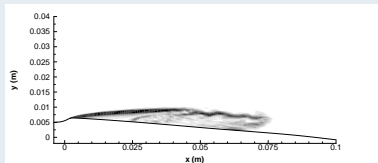
Comparison between Rebound and S-A $k-l$ models



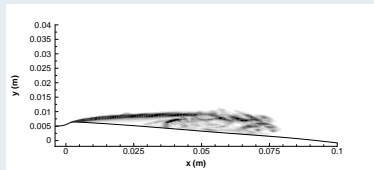
- time-averaged profiles quasi similar
- better pressure fluctuations with S-A model

Density gradient, Schlieren visualization

$k-l$ SAS



$k-l$ Rebound



Detached Eddy Simulation of Spalart

$$\frac{\partial \rho \tilde{\nu}}{\partial t} + \text{div} \left[\vec{V} \rho \tilde{\nu} - \frac{1}{\sigma} (\mu + \rho \tilde{\nu}) g \vec{\text{rad}} \tilde{\nu} \right] = c_{b1} (1 - f_{t2}) \tilde{S} \rho \tilde{\nu} + \frac{c_{b2}}{\sigma} g \vec{\text{rad}} \rho \tilde{\nu} \cdot g \vec{\text{rad}} \tilde{\nu} - \left(c_{\omega 1} f_{\omega} - \frac{c_{b1}}{\kappa^2} f_{t2} \right) \rho \frac{\tilde{\nu}^2}{\tilde{d}^2}$$

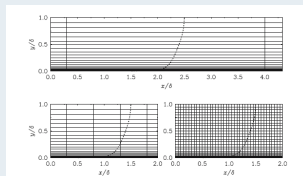
with $\tilde{d} = \min(d, C_{DES} \Delta)$ and $\Delta = \max(\Delta x, \Delta y, \Delta z)$.

C_{DES} is a constant evaluated for the decay of isotropic turbulence = 0.65

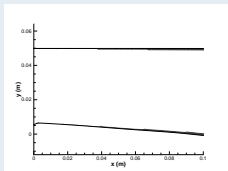
In equilibrium area : $\tilde{\nu} = C_{DES}^2 \Delta^2 S$

Drawbacks

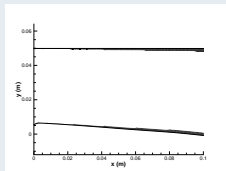
- Grid induced separation.
- Transition between RANS-mode and LES-mode : "grey" zone.
- Calibration of the constant C_{des} .



Influence of C_{DES} . RANS regions (black) and LES (white)

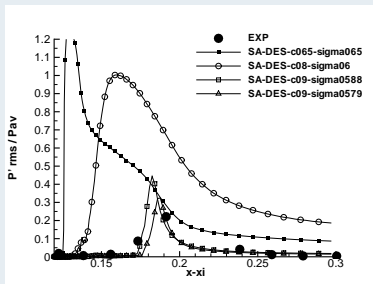
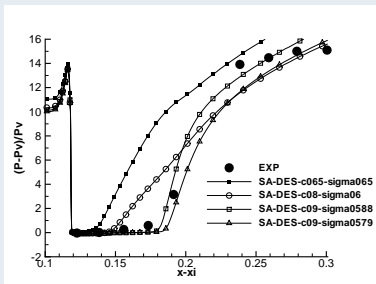


$C_{des} = 0.65$

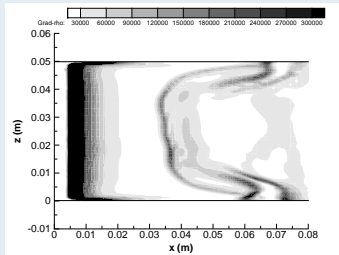
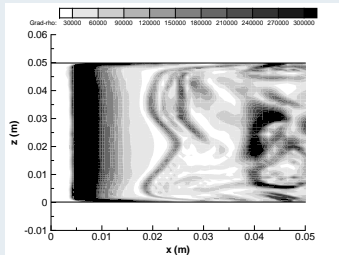
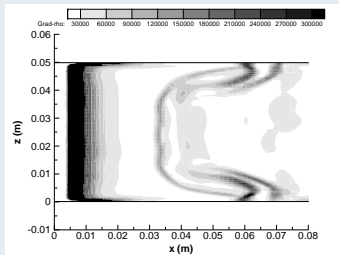
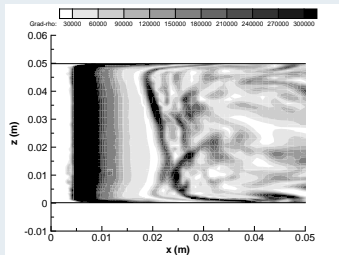


$C_{des} = 0.9$

Wall pressure and RMS fluctuations

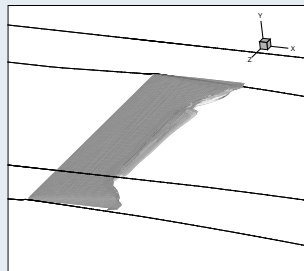
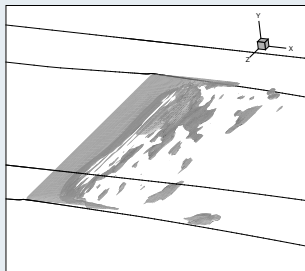
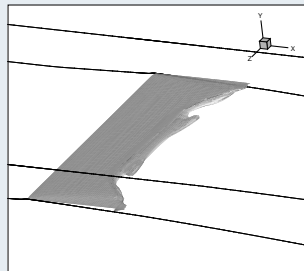
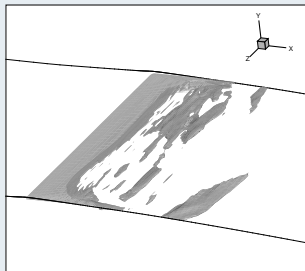


Comparison between $k - \ell$ S-A and DES simulations



gradient density visualization at two different instants : $k - \ell$ S-A (left) and DES (right)

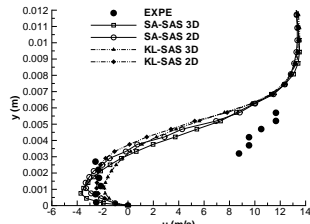
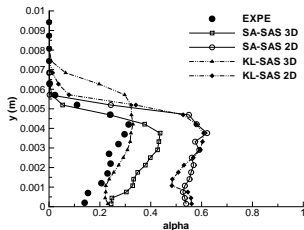
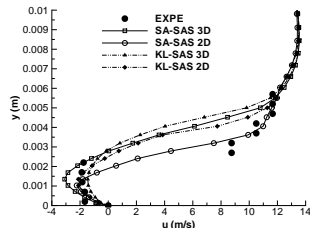
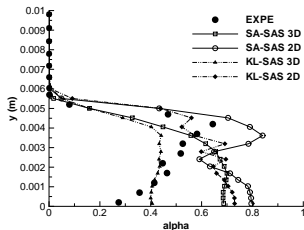
Comparison between $k - \ell$ Scale-Adaptive and DES simulations



Iso-surface of the void fraction for the value of 60% at two different

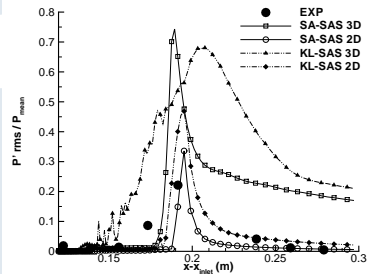
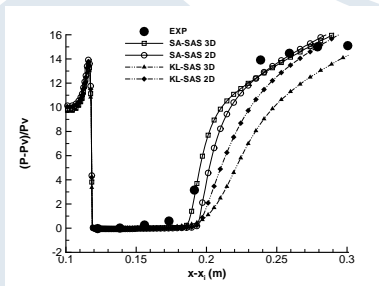
Comparison between 2D and 3D simulations

- Good agreement between models and experiment (mid-span profiles)
- 3D computations provides void fraction lower than 2D computations



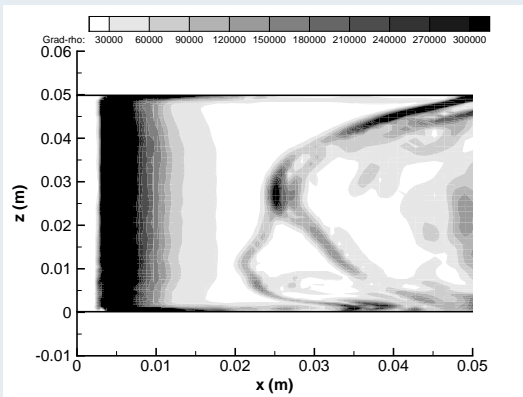
Comparison between 2D and 3D simulations

The level of RMS pressure fluctuations is largely overestimated by the 3D computations



Time-averaged wall pressure and RMS fluctuations

Dynamic behaviour - oblique mode using S-A simulations



Density gradient visualization

→ a transversal instability at a low frequency : 6 Hz.

One-fluid filtered equations and LES

Scales

- Micro-scales, scales which are small enough to describe individual bubble shapes.
- Meso-scales, which are comparable to bubble sizes.
- Macro-scales, which entail enough bubbles for statistical representation.
- When LES is applied at a micro-scale, combination with interface tracking methods (see Lakehal).
- When LES is applied at a macro-scale, the interface resolution is not considered.
- The scale separation is mathematically obtained by applying a convolution product using a large-scale-pass filter (function G).

Filter

- For a quantity ϕ , the filtered variable is defined as : $\overline{\phi} = G \circ \phi$
- The Favre filtered variable : $\tilde{\phi} = \overline{\rho\phi}/\overline{\rho}$
- The filtered phase indicator function $\overline{X_k} = G \circ X_k = \alpha_k$ can be interpreted as a filtered volume fraction of phase k .

Phase indicator function

The phase indicator function is defined as :

$$X_k(M(x, t)) = \begin{cases} 1 & \text{if phase } k \text{ is present in point } M(x, t) \text{ at } t \\ 0 & \text{otherwise} \end{cases}$$

Equations

Assumptions : the filtering operator commutes with time and spatial derivatives. The mass transfer is assuming to be proportional to the velocity divergence through a constant C .

$$\begin{aligned} \frac{\partial \bar{\rho}}{\partial t} + \frac{\partial (\bar{\rho} \tilde{u}_i)}{\partial x_j} &= 0 \\ \frac{\partial \bar{\rho} \tilde{u}_i}{\partial t} + \frac{\partial (\bar{\rho} \tilde{u}_i \tilde{u}_j + \bar{P} \delta_{ij})}{\partial x_j} &= \text{div} \left(2\bar{\mu} \tilde{S} + 2\tau_{\mu S} - 2\tau_{\rho S} - \tau_{\rho uu} \right) \\ \frac{\partial \bar{\rho} \tilde{E}}{\partial t} + \frac{\partial (\bar{\rho} \tilde{H} \tilde{u}_i + \bar{Q}_i^v)}{\partial x_j} &= \text{div} \left(2\tau_{\mu S u} - 2\bar{\mu} \tilde{V} \tau_{\rho S} - 2\bar{\mu} \tilde{S} \tau_{\rho u} \right) \\ &+ \text{div} \left(4\bar{\mu} \tau_{\rho u} \tau_{\rho S} + \bar{\rho} \tilde{H} \tau_{\rho u} - \tau_{\rho H u} \right) \\ \frac{\partial \alpha}{\partial t} + \tilde{V} \cdot \nabla \alpha - C \frac{\partial \tilde{u}_i}{\partial x_j} &= -\tau_{u\alpha} - C \text{div} (\tau_{\rho u}) \end{aligned}$$

Subgrid terms

$$\begin{aligned}
 \tau_{u\alpha} &= \overline{V \cdot \nabla X_v} - \tilde{V} \cdot \nabla \alpha & ; & & \tau_{\rho u} &= \tilde{V} - \overline{V} \\
 \tau_{\mu S} &= \overline{\mu S} - \bar{\mu} \bar{S} & ; & & \tau_{\rho S} &= \tilde{S} - \bar{S} \\
 \tau_{\rho H u} &= \overline{\rho H V} - \bar{\rho} \tilde{H} \tilde{V} & ; & & \tau_{\mu S u} &= \overline{\mu S V} - \bar{\mu} \bar{S} \bar{V} \\
 \tau_{\rho u u} &= \bar{\rho} \left(\widetilde{V \otimes V} - \tilde{V} \otimes \tilde{V} \right)
 \end{aligned}$$

- The subgrid term $\tau_{u\alpha}$ is specific to two-phase flows.
- The influence and the hierarchy of all these terms have never been investigated in cavitating flows.
- The magnitude of the different subgrid terms was *a priori* evaluated in the case of phase separation flows and turbulence bubble interaction (Labourasse, Vincent) \rightarrow the influence of $\tau_{u\alpha}$ is highly dependent on the flow configurations and/or on the chosen two-phase description.

Modelling difficulties

- Lots of assumption in models.
- Lack of experimental data or DNS.
- Compressibility turbulent closure due to the no divergence free fluctuating velocity field
- Advanced turbulence models and hybrid turbulence model to improve the level of resolved scales
- A challenge : LES in cavitating flows.